2nd Year Physics

Chapter # 12

Electrostatics

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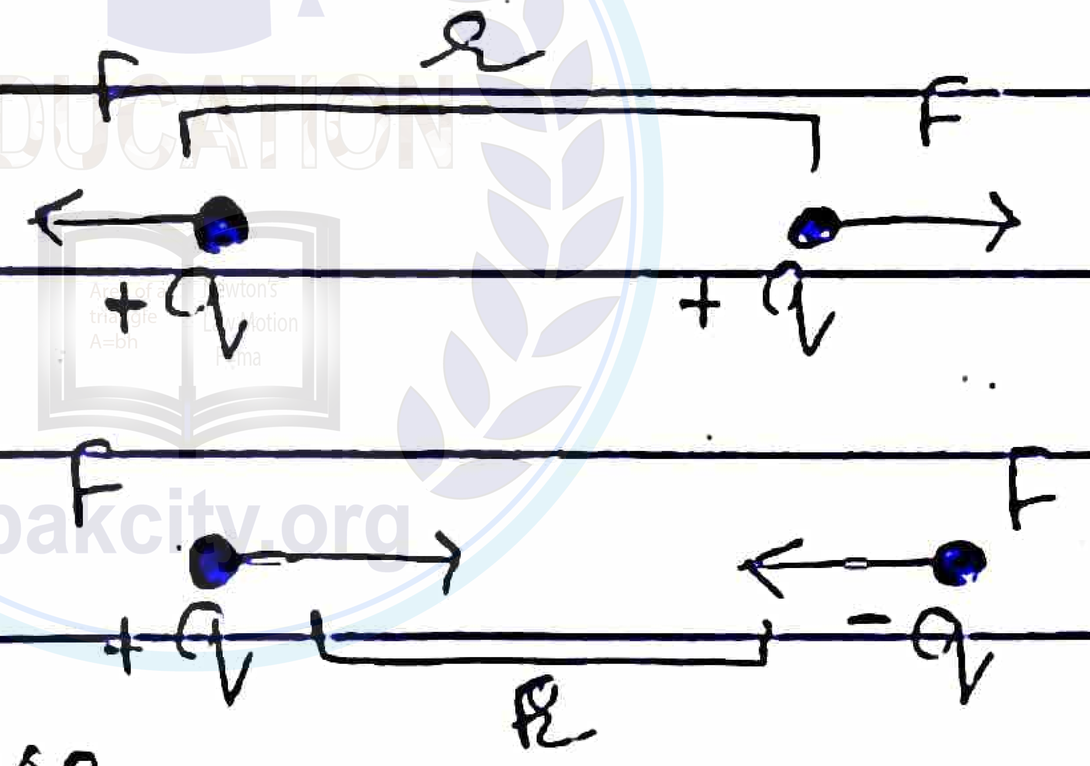
Coulomb's Law:

Statement:

"The force between two points charges is directly proportional to the product of magnitude of their charges and inversely proportional to the square of the distance between them"

Explanation:



Concider two charges q_1 and q_2 seperated by the distance r as shown in the figure. If the charges are same they will repel each other and if the charges are opposite they will attract each other. Both the charges exert equal and opposite forces on each other.

According to Coulomb's law:

$$F \propto q_1 q_2$$
$$F \propto \frac{1}{r^2}$$

By combining both the equations:

$$F \propto \frac{q_1 q_2}{r^2}$$

$$F = K \frac{q_1 q_2}{r^2}$$

Here 'K' is the constant called proportionality constant whose value depends on the medium between the charges. If there is no medium then the value of K is

$$K = \frac{1}{4\pi\varepsilon_0}$$

Here ε_o is the premitivity of force space and its value is

$$\boxed{\varepsilon_o = 8.85 \times 10^{-12}\ C^2N^{-1}m^{-2}}$$

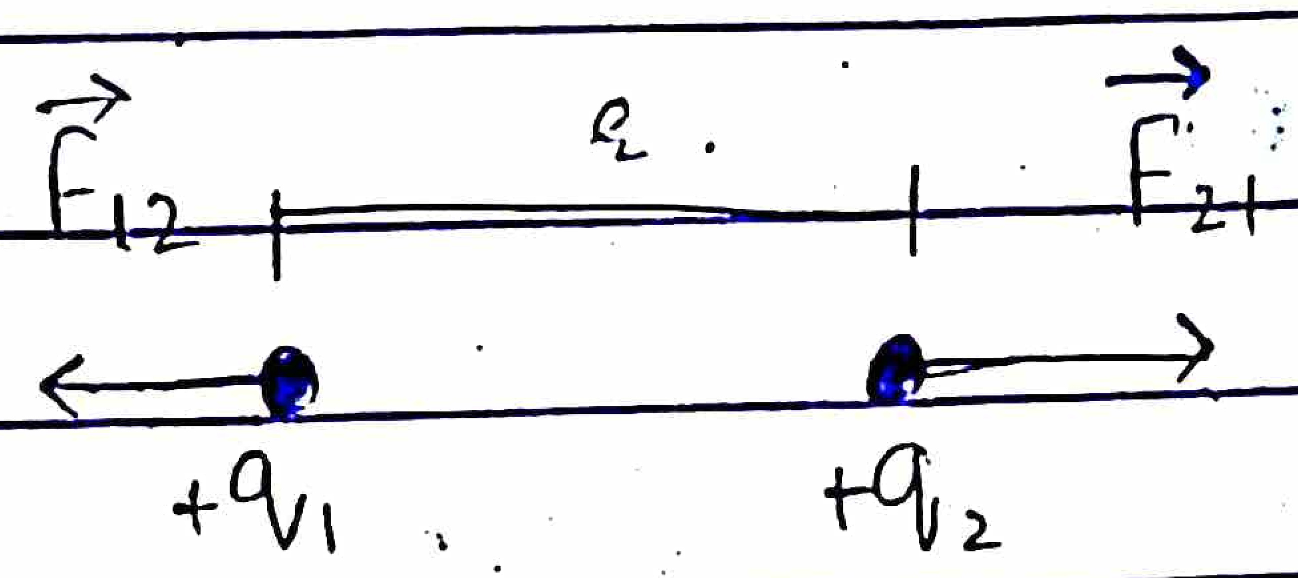
And K will become

$$\boxed{K = 9 \times 10^{9}\ Nm^2C^{-2}}$$

Now the Coulombs law will be written as.

$$K = \frac{1}{4\pi\varepsilon_o}\frac{q_1 q_2}{r^2}$$

Direction of force:



Both the charges ~~forces~~ exert equal and opposite forces on each other F_{12} is the force acting on charge q_1 by charge q_2, and F_{21} is the force acting on charge q_2 by the charge q_1.

Mathematically:

$$\vec{F}_{12} = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r^2}\hat{r}_{12}$$

$$\vec{F}_{21} = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r^2}\hat{r}_{21}$$

Here, $\hat{r}_{12}$ and $\hat{r}_{21}$ is the unit vectors that show the direction of the force $\vec{F}_{12}$ and $\vec{F}_{21}$ respectively

$$\vec{F}_{12} = -\vec{F}_{21}$$

Effect of medium:

If the dielectric medium is placed between the charges, the electrostatic force will be decreased by a factor ε_r.

Mathematical:

$$F' = \frac{1}{4\pi\varepsilon_0\varepsilon_r}\frac{q_1q_2}{r^2}.$$

Relative Permitivity.

$$\frac{F}{F'} = \frac{\cancel{\frac{1}{4\pi\varepsilon_0}}\cancel{\frac{q_1q_2}{r^2}}}{\cancel{\frac{1}{4\pi\varepsilon_0\varepsilon_r}}\cancel{\frac{q_1q_2}{r^2}}}$$

$$\frac{F}{F'} = \frac{1}{1/\varepsilon_r}$$

$$\frac{F}{F'} = \varepsilon_r$$

Relative permitivity is the ratio of electrostatic force when there is no medium between the charges to the force when there is dielectric medium between them.

→ There is no medium of ε_r.

Electric Field:

The area or region around a charge in which it can exert its electrostatic force of attraction or repulsion on the other charge is called electric field.

The concept of electric field was firstly given by michael Faraday.

Electric Field Intensity / Electric field Strength:

The strength of electric field is called electric field intensity.

$\boxed{OR}$

The electric force acting per unit charge is called electric field intensity. It is denoted by $\vec{E}$. mathematically,

$$\vec{E} = \frac{\vec{F}}{q_o}$$

Electric intensity is a vector quantity. Its unit is NC^{-1}.

Direction: The direction of electric intensity is same as the direction of electric force.

For two charges q and q_o, the electric force will be:

$$F = \frac{1}{4\pi\varepsilon_o}\frac{q q_o}{r^2}$$

And

$$E = \frac{F}{q_o}$$

$$E = \frac{\frac{1}{4\pi\varepsilon_o}\frac{q \cancel{q_o}}{r^2}}{\cancel{q_o}}$$

$$E = \frac{1}{4\pi\varepsilon_o}\frac{q}{r^2}$$

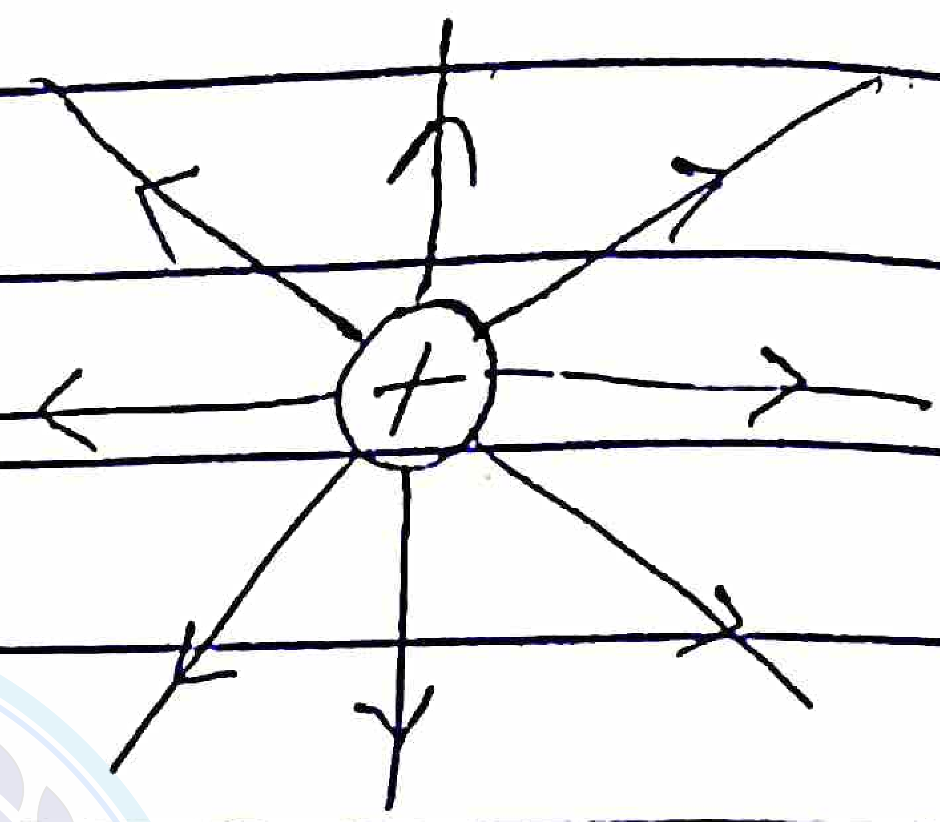
In vector form.

$$\vec{E} = \frac{1}{4\pi\varepsilon_o}\frac{q}{r^2}\hat{r}$$

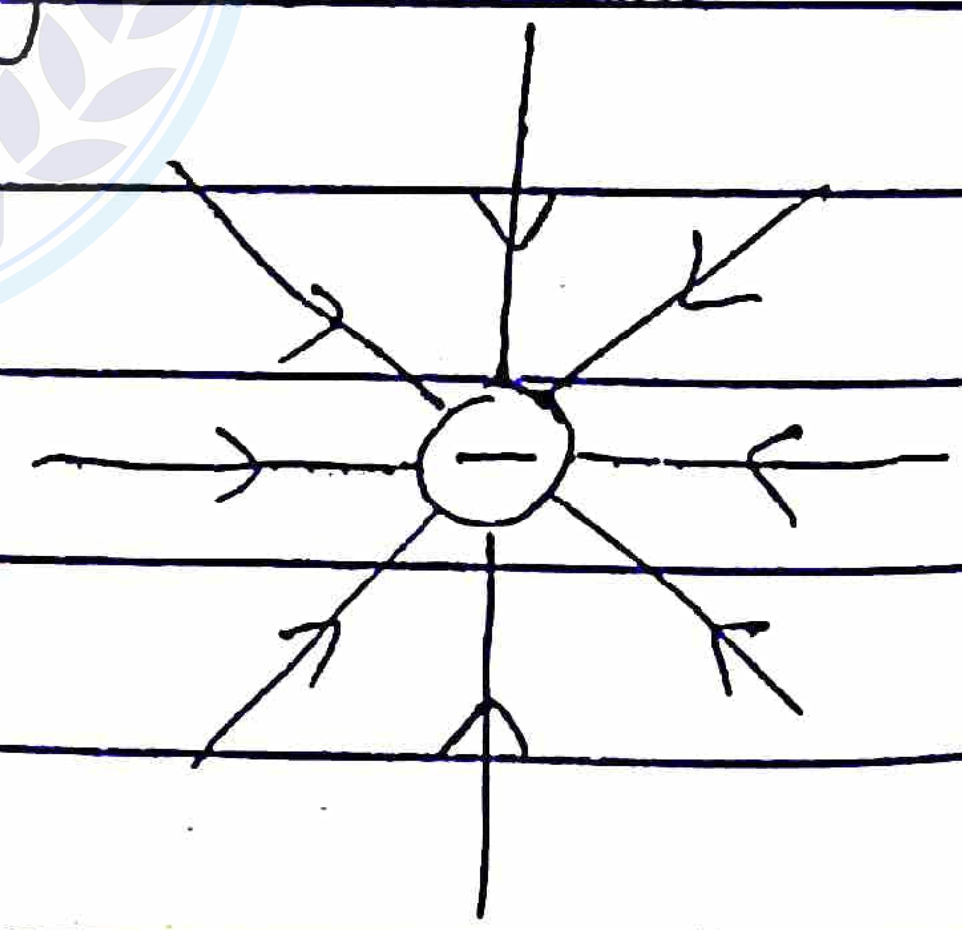
Electric Field Lines:

The imaginary lines that are used to show the direction and strength of electric field around a charge are called electric field lines.

Properties:

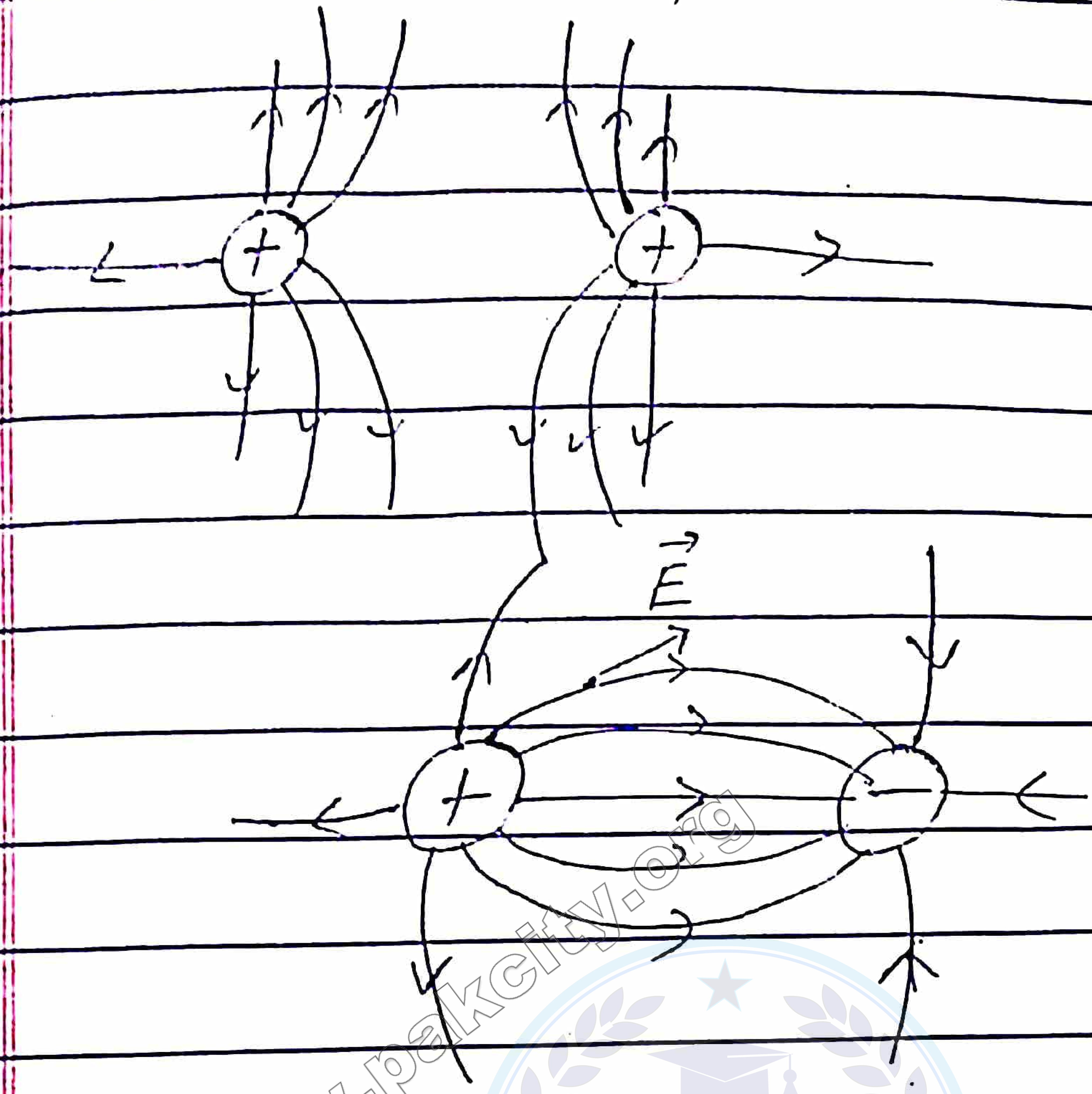


i- Electric field lines are directed outward from the positive charge.



ii- Electric field lines are directed inward to the negative charge.

iii- Electric field lines are close where the field is strong while electric field lines are far apart where the field is weak.



iv- The tangent to electric field lines show the direction of electric intensity.

v- Electric field lines never cross each other because every line has its own specific direction.

Electric Flux:
The number of electric field lines passing through a certain area is called electric flux

OR

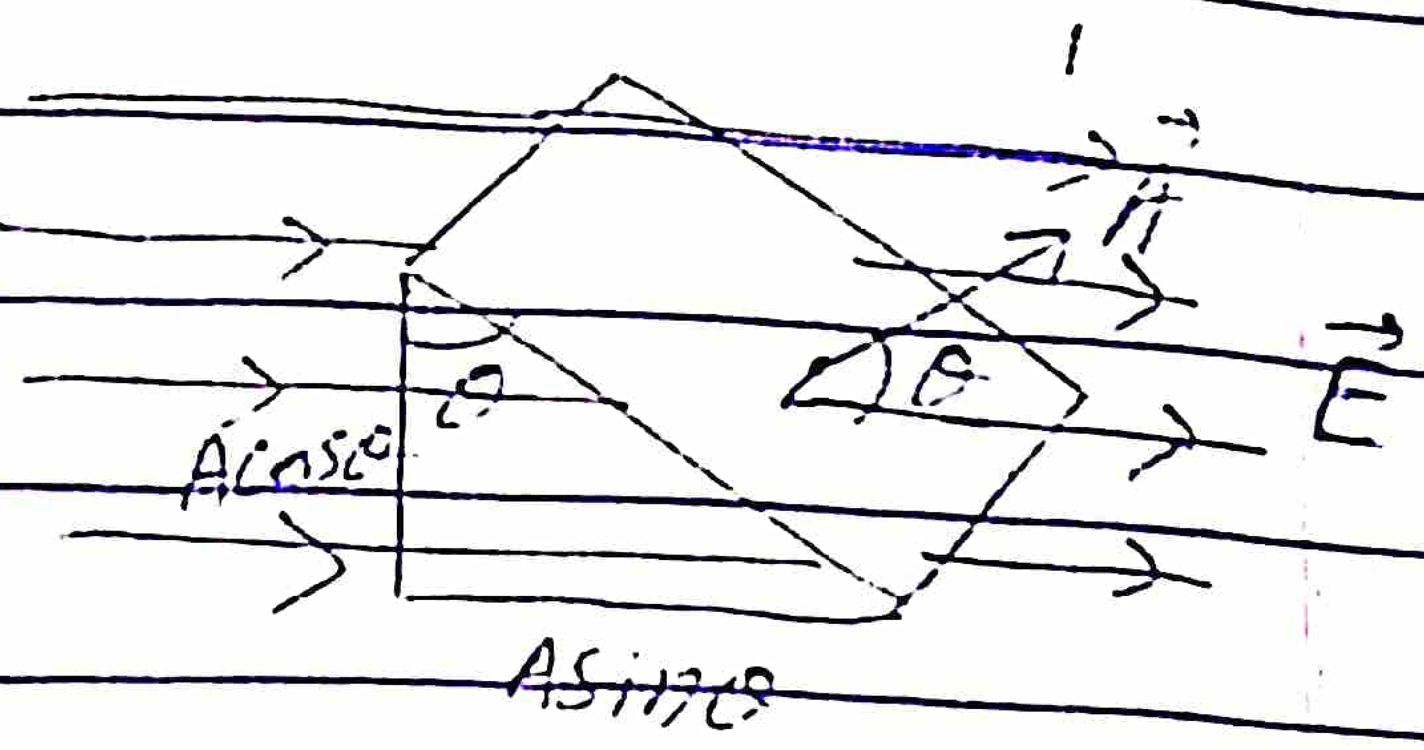
The dot product of electric intensity $\vec{E}$ and vector area $\vec{A}$ is called electric flux ϕ. mathematically,

$$\phi = \vec{E} \cdot \vec{A}$$

The electric flux ϕ is a scalar quantity and its unit is Nm^2C^{-1}

Vector Area: The line normal to the surface is called vector area. The magnitude of the vector area is the value of area of the surface and direction perpendicular to the surface

Explanation:
Consider a surface with vector area $\vec{A}$ placed in

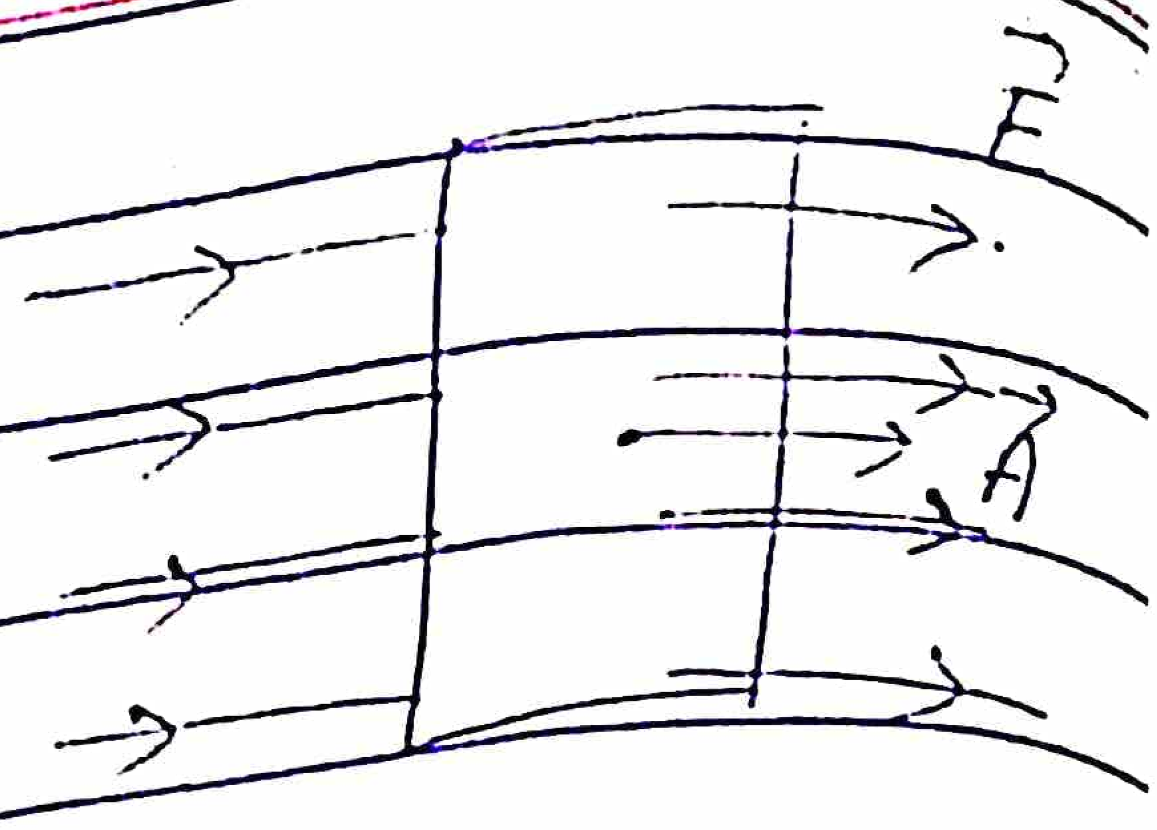


the path of electric field lines. The angle between electric field lines and vector area is θ as shown in the figure. Then we resolve the vector area into its rectangular components $A\sin\theta$ and $A\cos\theta$. The electric field lines are passing through the component $A\cos\theta$. So,

$$\phi = E(A\cos\theta)$$

$$\phi = EA\cos\theta$$

Maximum Flux: The electric flux through the surface will be maximum when the surface is held perpendicular to the electric field lines.



In this way. electric field lines will be parallel to the vector area.

$$\phi = EA\cos\theta$$

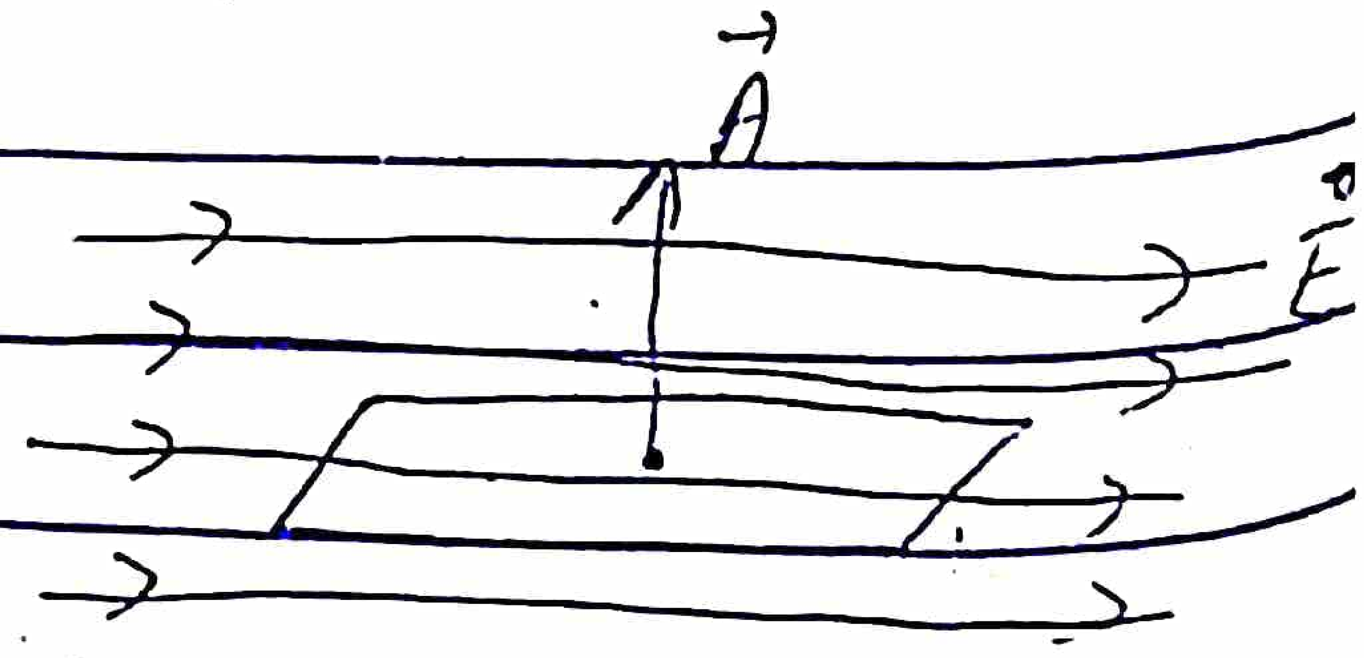
here $\theta = 0^\circ$

$$\phi = EA\cos 0^\circ$$

$$\phi_{max.} = EA$$

Minimum Flux:

The electric flux through the surface will be minimum when the surface is held parallel to the electric field lines.

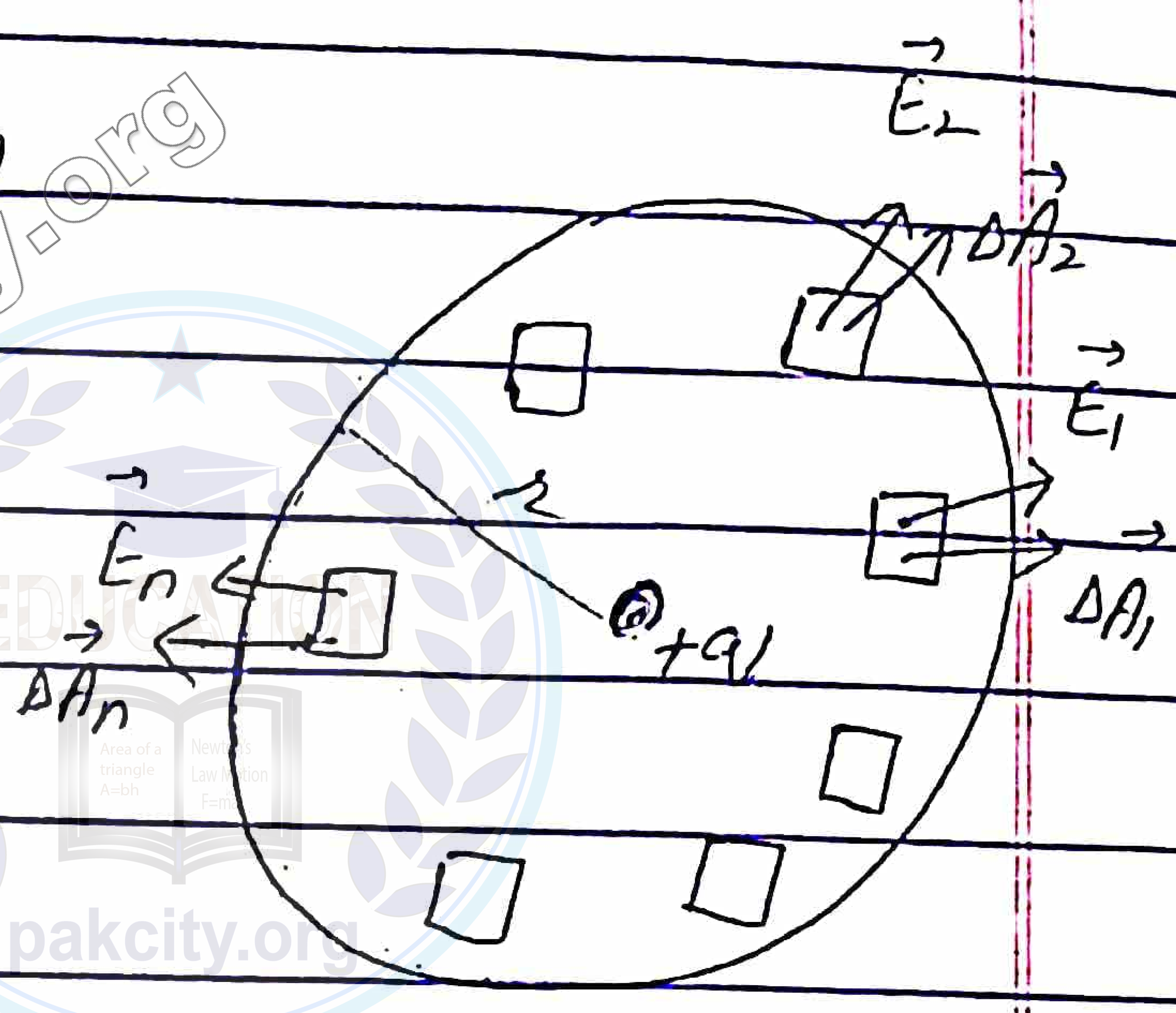


In this way. electric field lines will be perpendicular to vector area

$$\phi = EA\cos 90^\circ$$
$$= EA(0)$$
$$\phi_{min.} = 0$$

Electric Flux Through A Surface Enclosing A Charge:

Consider a closed surface in the form of a sphere of radius r as shown in the figure.



A positive charge is placed at the centre of the sphere through which electric field lines are directed outward. Then we divide the surface of sphere into n number of small pieces having vector areas $\Delta\vec{A}_1, \Delta\vec{A}_2, \ldots \Delta\vec{A}_n$

with electric intensities
$\vec{E}_1, \vec{E}_2, ---- \vec{E}_n$ respectively.
The total electric flux will be equal to the sum of electric flux through all the pieces.

$$\Phi_e = \Phi_1 + \Phi_2 + --- + \Phi_n$$

$$= \vec{E}_1 \cdot \Delta\vec{A}_1 + \vec{E}_2 \cdot \Delta\vec{A}_2 + --- -- + \vec{E}_n \cdot \Delta\vec{A}_n$$

$$\Phi_e = E_1 \Delta A_1 \cos\theta_1 + E_2 \Delta A_2 \cos\theta_2 + --- + E_n \Delta A_n \cos\theta_n$$

The electric intensity and vector area are parallel for all the pieces. So,

$$\theta_1 = \theta_2 = --- = \theta_n = 0^\circ$$

$$\cos\theta_1 = \cos\theta_2 = --- = \cos\theta_n = 1$$

So,

$$\Phi_e = E_1 \Delta A_1 + E_2 \Delta A_2 + --- + E_n \Delta A_n$$

All the pieces on the surface of the sphere are at the same distance from the charge. So, the electric intensity will be same for all the pieces.

$$|E_1| = |E_2| = \cdots = |E_n| = E$$

now

$$\Phi_E = E\Delta A_1 + E\Delta A_2 + \cdots + E\Delta A_n$$

$$= E(\Delta A_1 + \Delta A_2 + \cdots + \Delta A_n)$$

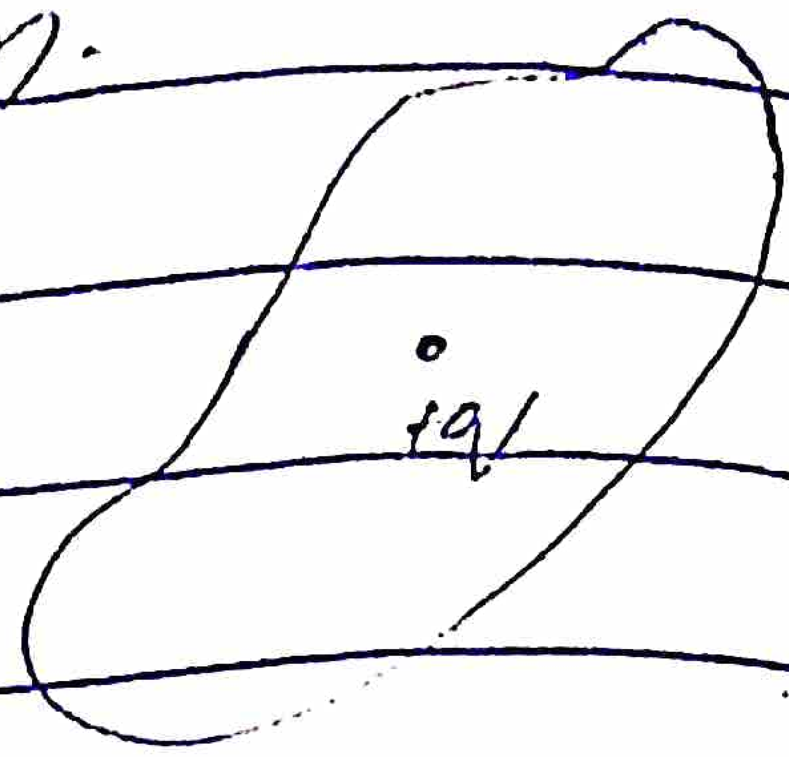
$$\Phi_E = E(\text{total spherical surface area})$$

$$= \frac{1}{4\pi\varepsilon_0}\frac{q}{\cancel{r^2}}(4\pi \cancel{r^2})$$

$$\Phi_E = \frac{q}{\varepsilon_0}$$

Dependance.

The electric flux through a surface depends upon the magnitude of charge and medium of the surface. It does not depend upon the shape of the closed surface. The shape may be of any pallcon.



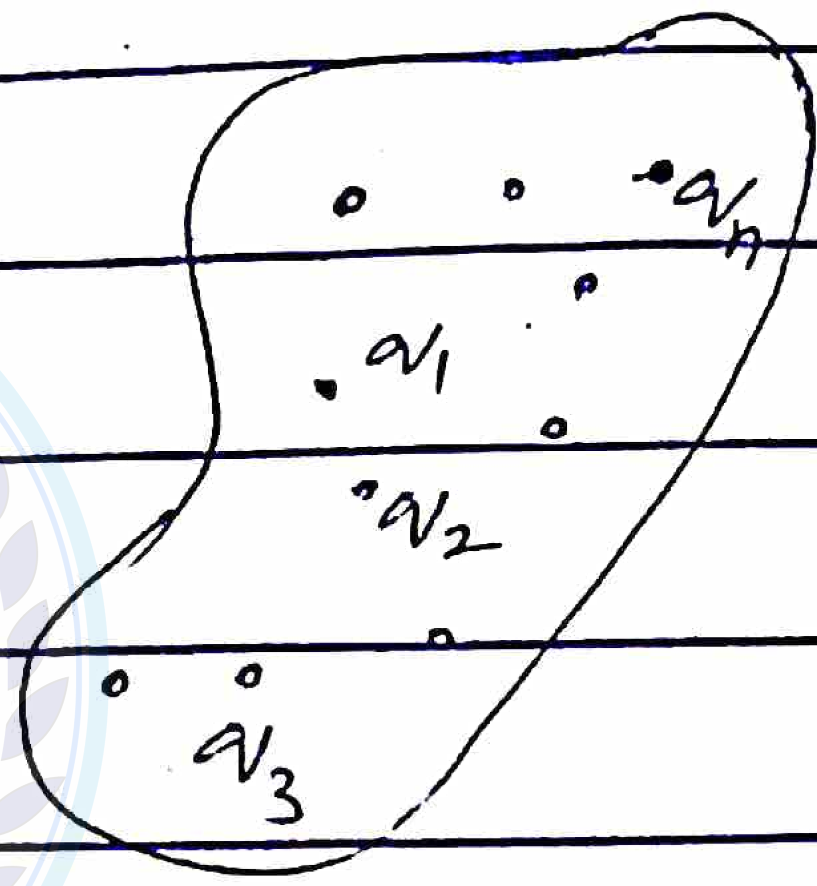
Gauss's Law:

Statement:

"The flux through any closed surface is $\frac{1}{\epsilon_0}$ times the total charge enclosed in it"

Explanation:

Consider a closed surface of irregular shape in which charges $q_1, q_2, \ldots q_n$ are placed arbitrary as shown in the figure.



The total flux through the surface will be equal to the sum of electric flux for all the charges.

$$\Phi_e = \Phi_1 + \Phi_2 + \Phi_3 + \cdots + \Phi_n$$

$$\phi_e = \frac{q_1}{\varepsilon_0} + \frac{q_2}{\varepsilon_0} + \frac{q_3}{\varepsilon_0} + \cdots + \frac{q_n}{\varepsilon_0}$$

$$= \frac{1}{\varepsilon_0}(q_1 + q_2 + q_3 + \cdots + q_n)$$

$$= \frac{1}{\varepsilon_0}(\text{total charge enclosed by closed surface})$$

$$\phi_e = \frac{1}{\varepsilon_0}(Q)$$

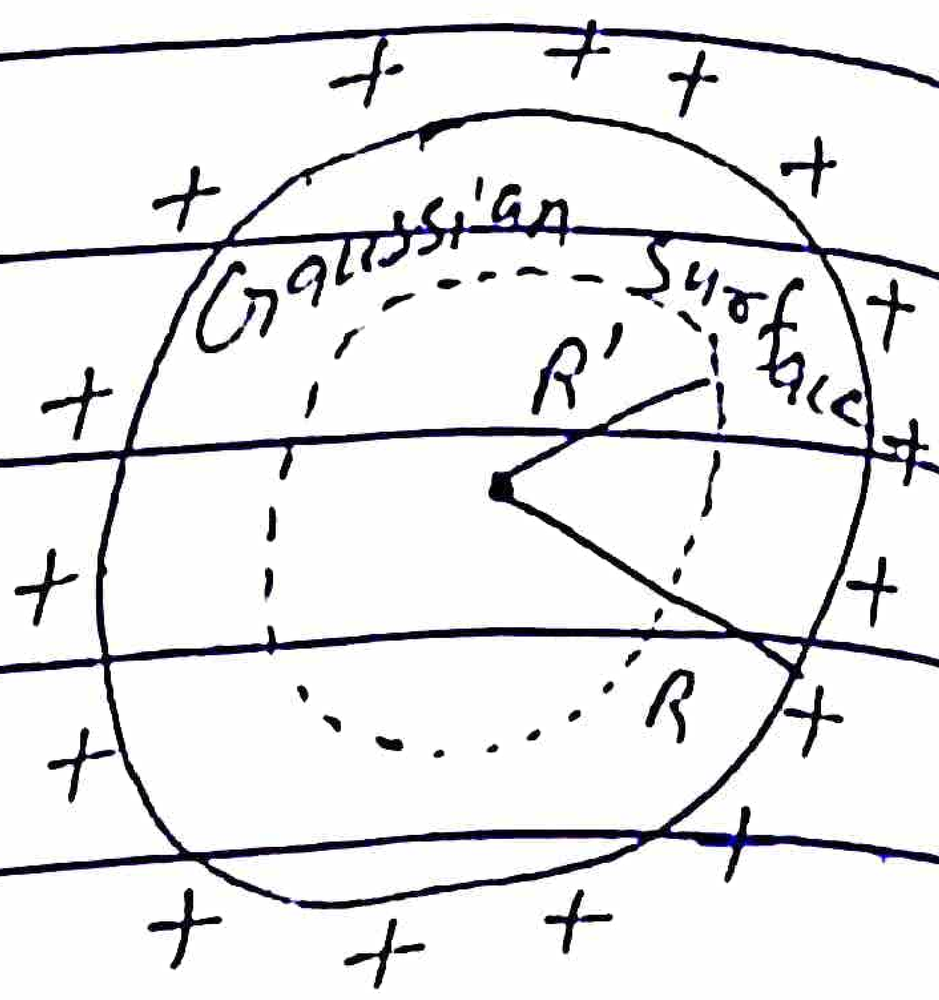
This is the mathematical expression for the Gauss's law.

Gaussian Surface:

The imaginary closed surface that passes through a point at which Gauss's law is applied to calculate the electric intensity is called Gaussian surface.

Intensity of Field Inside A Hollow Charged Sphere:

Consider a sphere of radius R on which positive charge is uniformly distributed as shown in the figure. Then we take a Gaussian surface inside the sphere of radius R'. According to Gauss's law:



$$\phi_e = \frac{1}{\varepsilon_o} Q$$

There is no charge inside the Gaussian surface.

$$Q = 0$$

So,

$$\phi_e = \frac{1}{\varepsilon_o}(0)$$

$$\phi_e = 0 \longrightarrow (1$$

We know that

$\phi = \vec{E} \cdot \vec{A}$ (2

Comparing eq. (1 and eq. (2

$$\vec{E} \cdot \vec{A} = 0$$

either $\vec{E} = 0$ OR $\vec{A} = 0$

But the area of the sphere could not be zero. So,

$$\vec{E} = 0$$

Result:

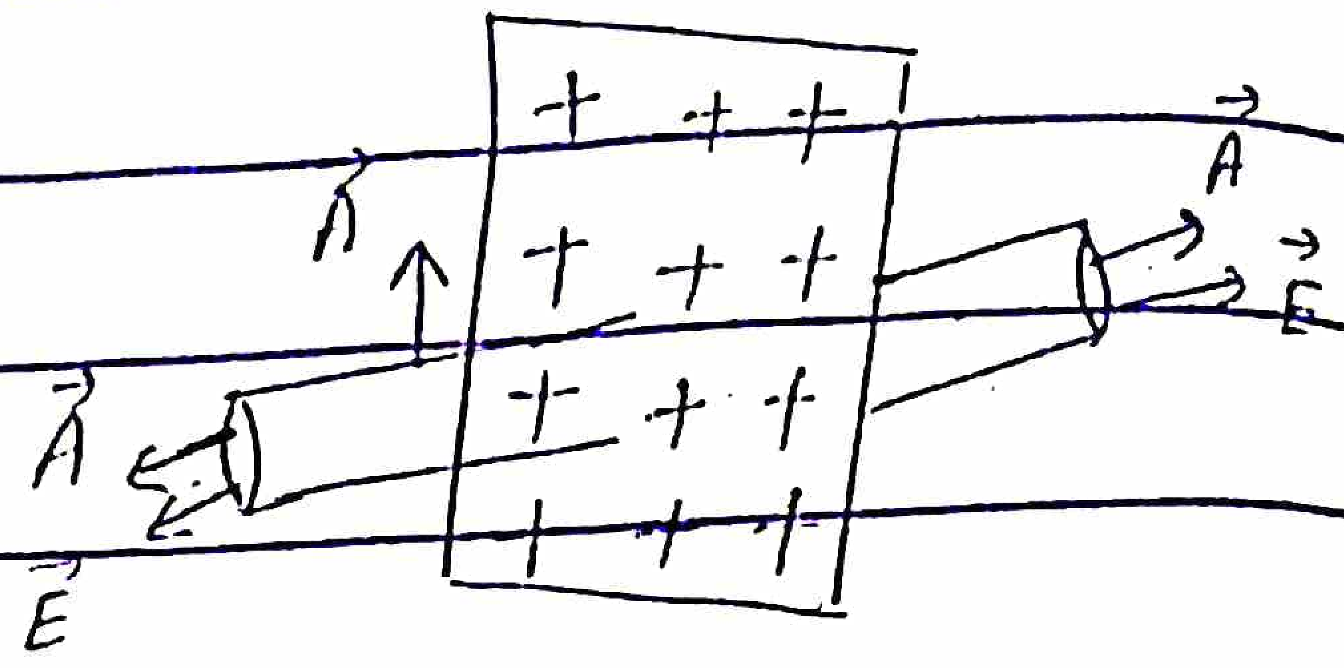
The electric intensity is zero inside a hollow charged sphere.

$\boxed{\text{OR}}$

The interior of a hollow charged metal sphere is a field free region.

Electric Intensity Due to An Infinite Sheet of Charge:

Consider a part of an infinite sheet over which positive charge is uniformly distributed



as shown in the figure. We take Gaussian surface in the form of a cylinder that passes through the sheet. There are three main parts of the Gaussian surface, two are its faces and one is its inner part. Now we will calculate the electric flux through these three parts.

For Outer faces: The electric field lines are parallel to the vector area for the faces. So,

$$\phi_1 = \phi_2 = \vec{E}.\vec{A}$$
$$= EA\cos\theta$$
$$= EA\cos 0^\circ$$

$$\phi_1 = \phi_2 = EA$$

For inner Part: The electric field lines are perpendicular to vector area for the inner part so,

$$\phi_3 = EA\cos\theta$$
$$= EA\cos 90^\circ$$
$$= EA(0)$$
$$\phi_3 = 0$$

The total electric flux will be:

$$\phi_e = \phi_1 + \phi_2 + \phi_3$$

$$= EA + EA + 0$$

$$\phi_e = 2EA \longrightarrow ①$$

According to Gauss's law

$$\phi_e = \frac{1}{\varepsilon_0}(Q)$$

The electric charge per unit area is called the surface charge density. It is denoted by σ.

$$\sigma = \frac{Q}{A}$$

$$\Rightarrow \quad Q = \sigma A$$

Now

$$\phi_e = \frac{1}{\varepsilon_o}(\sigma A) \rightarrow (2$$

Comparing eq. (1 and eq. (2

$$2E\cancel{A} = \frac{1}{\varepsilon_o}(\sigma \cancel{A})$$

$$2E = \frac{\sigma}{\varepsilon_o}$$

$$E = \frac{\sigma}{2\varepsilon_o}$$

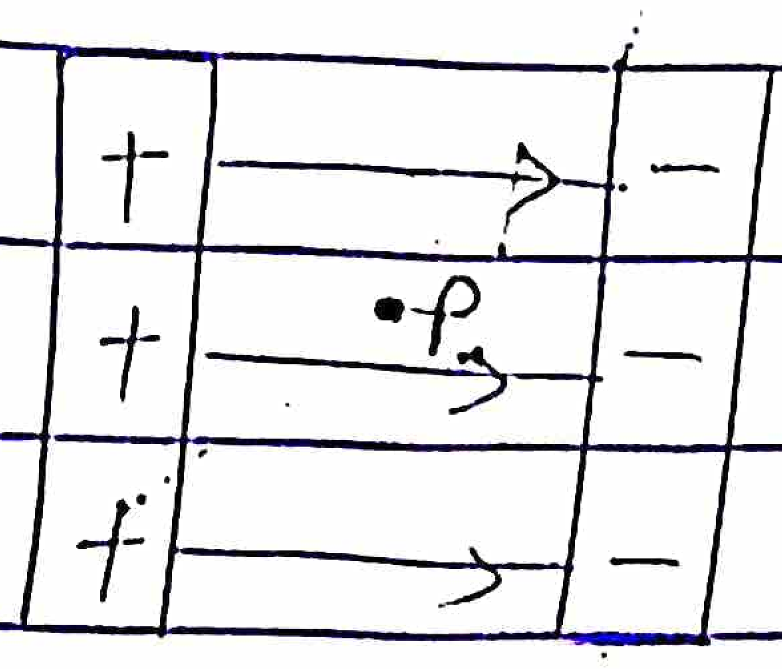
In vector form,

$$\vec{E} = \frac{\sigma}{2\varepsilon_o}\hat{r}$$

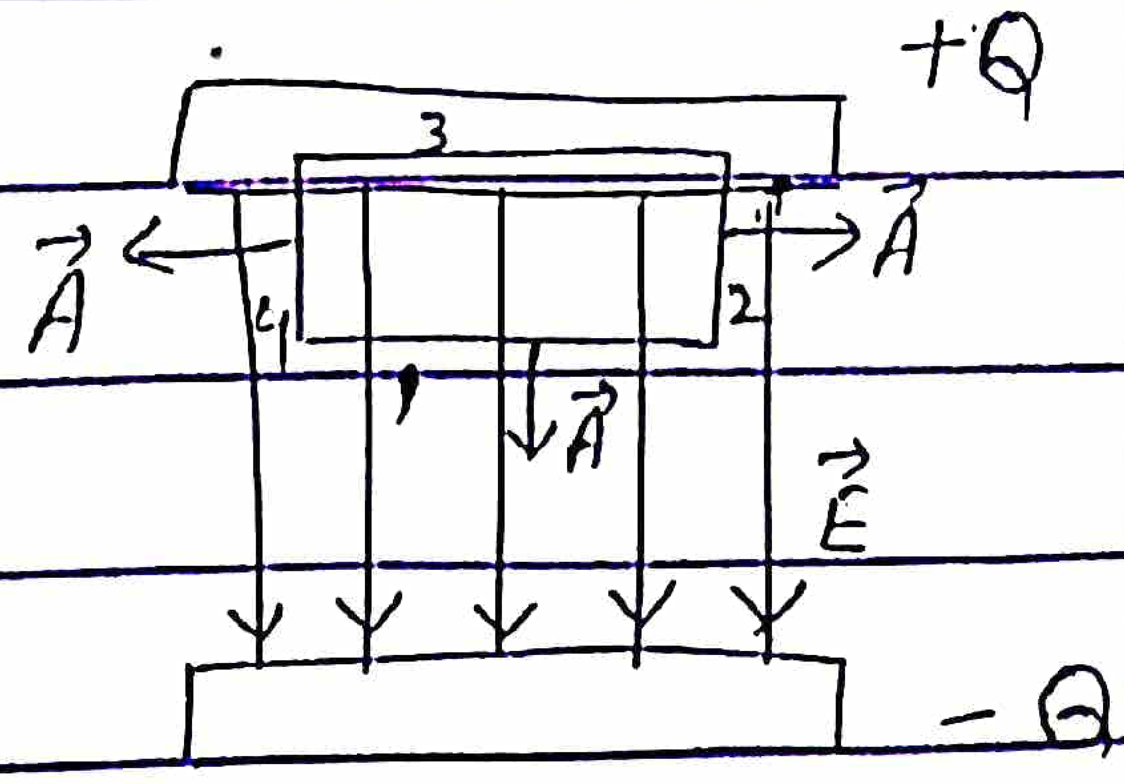
Here $\hat{r}$ is the unit vector that shows the direction of electric intensity that is directed away from the sheet.

Electric Intensity Between Two Oppositely Charged Parallel Plates:

Consider two parallel plates that are oppositely charged. One plate has positive charge and other have negative charge. we have to find the electric intensity between these plates.



We consider Gaussian surface in the shape of a rectangular box such that the upper side of box is in the positive plate. Now we have four sides of



Gaussian surface at which electric flux is to be determined.

For side.1 The electric field lines are parallel to vector area for this side. So,

$$\phi_1 = EA\cos\theta$$

here $\theta = 0°$

$$= EA\cos 0°$$

$$\phi_1 = EA$$

For side.2 The electric field lines are perpendicular to the vector area for this side. So,

$$\phi_2 = EA\cos\theta$$

here $\theta = 90°$

$$\phi_2 = EA\cos 90°$$

$$= EA(0)$$

$$\phi_2 = 0$$

For side.3 There is no electric field for this side.

$$\phi_3 = EA\cos\theta$$

here $E = 0$

$\Phi_3 = (0) A \cos\theta$

$\Phi_3 = 0$

For side 4 The electric field lines are perpendicular to the vector area for this side. So,

$$\Phi_4 = EA \cos\theta$$

here $\theta = 90^\circ$

$$= EA \cos 90^\circ$$

$$= EA(0)$$

$$\Phi_4 = 0$$

Now the total electric flux will be,

$$\Phi_e = \Phi_1 + \Phi_2 + \Phi_3 + \Phi_4$$

$$= EA + 0 + 0 + 0$$

$$\Phi_e = EA \rightarrow (1)$$

According to Gaussian's ~~surface~~ law

$$\Phi_e = \frac{1}{\epsilon_0} Q.$$

The electric charge per unit area is called surface charge density and it is denoted by σ

$$\sigma = \frac{q}{A}$$

$$q = \sigma A$$

Now

$$\Phi_e = \frac{1}{\varepsilon_0}(\sigma A) \rightarrow (2)$$

comparing eq (1) and eq (2)

$$E\cancel{A} = \frac{1}{\varepsilon_0}(\sigma \cancel{A})$$

$$E = \frac{\sigma}{\varepsilon_0}$$

In vector form:

$$\vec{E} = \frac{\sigma}{\varepsilon_0}\hat{r}$$

Here $\hat{r}$ is unit vector that shows the direction of electric intensity that is directed from positive plate to negative plate.

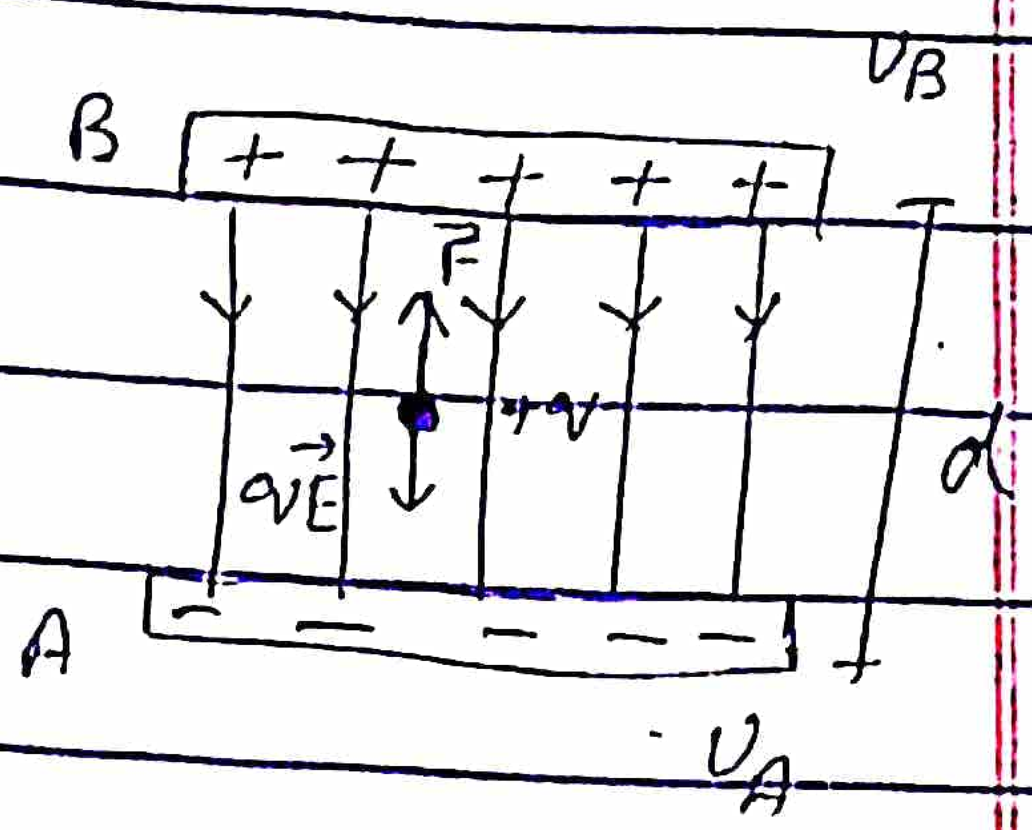
electroencephalography → brain.

ECG → heart.

electroretinography → eye.

Electrostatic Equilibrium:

When a charge moves in a potential difference with uniform velocity then it is said to be in electrostatic equilibrium.



Electric Potential Difference:

The potential difference between the two points can be defined as the difference ~~between~~ of the potential energy per unit charge.

→ The P.D b/w two points A and B in an electric field is defined as the work done in carrying a unit +ive charge from A to B while keeping the charge in equilibrium:

$$\Delta V = V_B - V_A = \frac{W_{AB}}{q_o} = \frac{\Delta U}{q_o}$$

Volt: A potential difference of 1 volt exists between two points if work done is one joul in moving a unit positive charge from one point to other, keeping equilibrium.

$$1 \text{ volt} = 1 \text{ joul} / 1 \text{ coulomb}$$

Electric Potential

The E.P at any point in an electric field is equal to work done in bringing a unit +ive charge from infinity to that point keeping it in equilibrium.

$$V = W/q_o$$

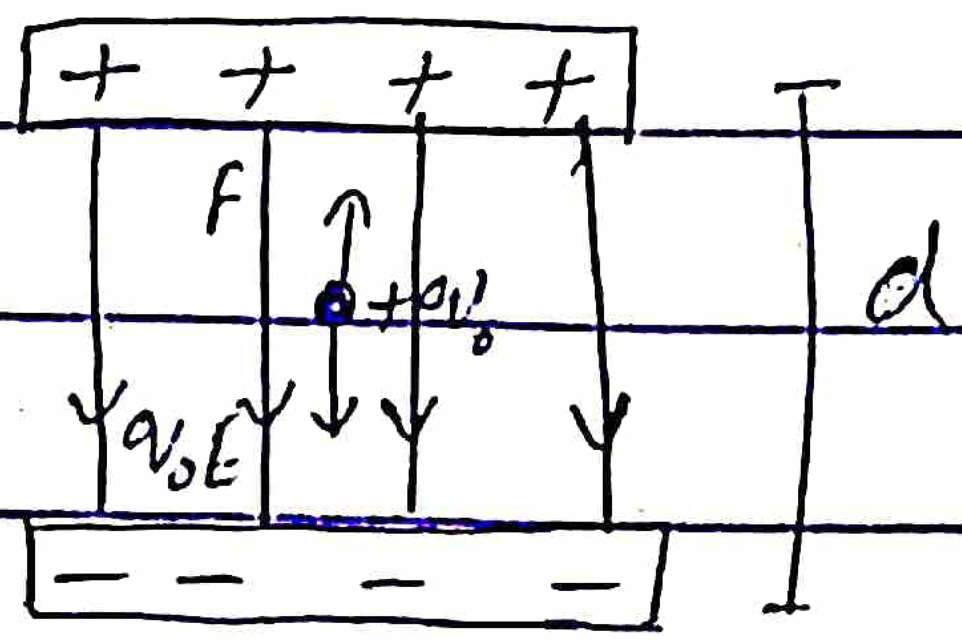
Potential Gradient:

The term $\frac{\Delta V}{\Delta r}$ is called potential gradient that gives the maximum rate of change of potential with distance.

We know that potential difference is given by:

$$\Delta V = \frac{W_{AB}}{q_o}$$

$$\Delta V = \frac{Fd\cos\theta}{q_o}$$



here $F = q_oE$

$\theta = 180°$

$$\Delta V = \frac{\cancel{q_o} Ed\cos 180°}{\cancel{q_o}}$$

$$\Delta V = Ed(-1)$$

$$\Delta V = -Ed$$

$$-\frac{\Delta V}{d} = E$$

$$E = -\frac{\Delta V}{d}$$

if

$$d = \Delta r$$

$$E = -\frac{\Delta V}{\Delta r}$$

Therefore, negative of potential gradient is equal to electric intensity.

Prove that $1\,Vm^{-1} = 1\,NC^{-1}$

$$1\,Vm^{-1} = \frac{1\,\text{volt}}{1\,\text{meter}}$$

$$= \frac{1\,\text{joule}}{1\,\text{coulomb} \times 1\,\text{meter}}$$

$$= \frac{1\,\text{newton} \times \cancel{1\,\text{meter}}}{1\,\text{coulomb} \times \cancel{1\,\text{meter}}}$$

$$= \frac{1\,\text{newton}}{1\,\text{coulomb}}$$

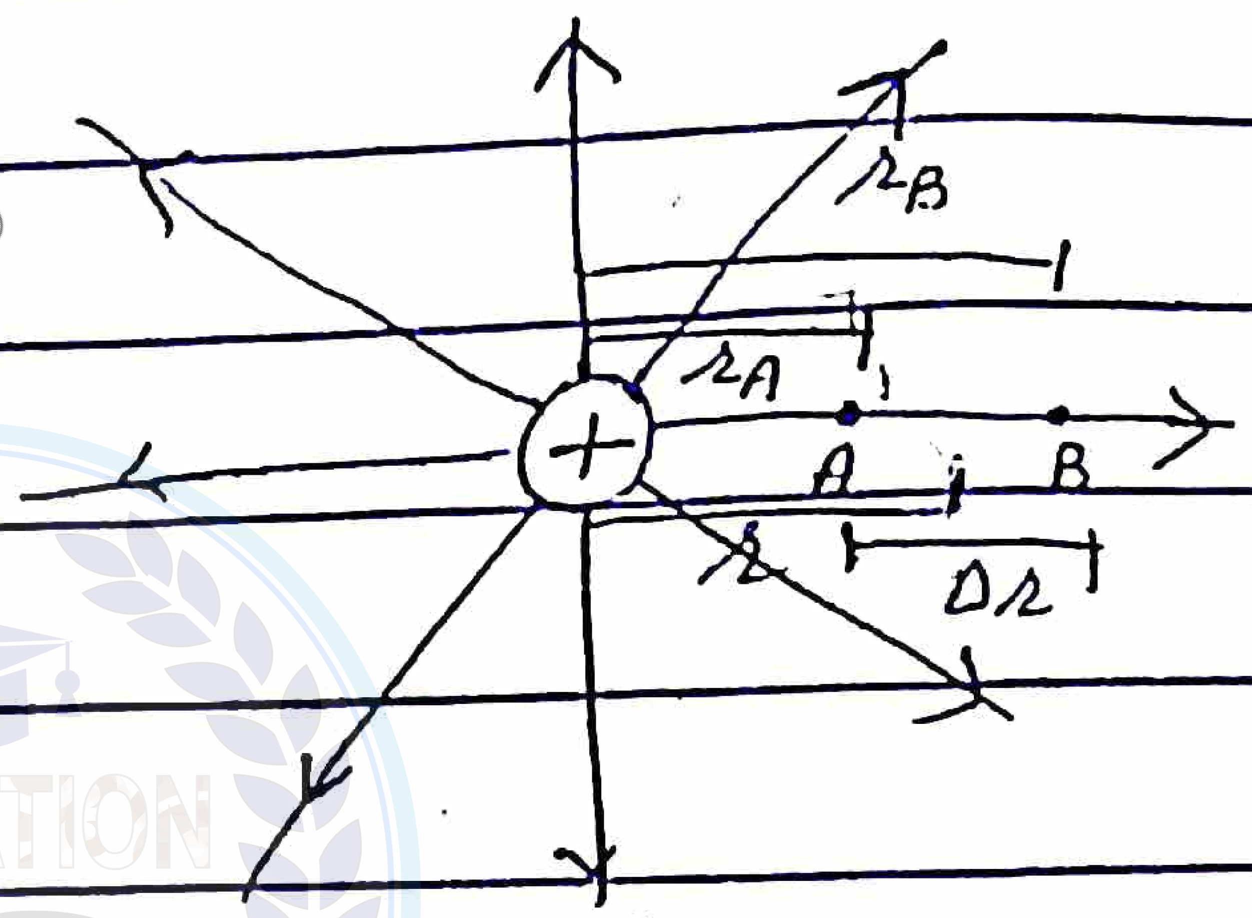
$$1\,Vm^{-1} = 1\,NC^{-1}$$

Electric Potential At A Point Due To A Point Charge:

Defination: Electric potential at a point in an electric field is equal to work done in bringing a unit positive charge from infinity to that point keeping it in equilibrium

Explanation:

Consider an electric field of positive charge in which an other positive charge is moved from point B to point A as shown in the figure. The distance of point A from the charge is r_A and that of point B is r_B. While Δr is the distance between points A and B. mathematically



$$\Delta r = r_B - r_A$$

$$r_B = \Delta r + r_A$$

We take a mid point of points A and B having distance r from the charge.

$$r = \frac{r_A + r_B}{2}$$

The electric intensity at this point will be:

$$E = \frac{1}{4\pi\varepsilon_0} \frac{q}{r^2}$$

The points A and B are very close to each other. They have approximately same distance from the charge.

$$r_A \; r_B = r \; r$$

$$r_A r_B = r^2$$

Now the expression for electric intensity will be:

$$E = \frac{1}{4\pi\varepsilon_0} \frac{q}{r_A r_B}$$

when the charge is moved from point B to point A. Then according to defination of potential gradient.

$$\Delta V = -E \Delta r$$

$$V_A - V_B = -E(r_A - r_B)$$

$$V_A - V_B = E(r_B - r_A)$$

$$= \frac{1}{4\pi\varepsilon_0} \frac{q}{r_A r_B}(r_B - r_A)$$

$$= \frac{q}{4\pi\varepsilon_0}\left(\frac{r_B - r_A}{r_A r_B}\right)$$

$$= \frac{q}{4\pi\varepsilon_0}\left(\frac{\cancel{r_B}}{r_A \cancel{r_B}} - \frac{\cancel{r_A}}{\cancel{r_A} r_B}\right)$$

$$V_A - V_B = \frac{q}{4\pi\varepsilon_0}\left(\frac{1}{r_A} - \frac{1}{r_B}\right)$$

For the electric potential point B is taken at infinity. so,

$$V_B = 0 \quad , \quad r_B = \infty$$

$$\frac{1}{r_B} = \frac{1}{\infty} = 0$$

so, $V_A - 0 = \frac{q}{4\pi\varepsilon_o}\left(\frac{1}{r_A} - 0\right)$

$$V_A = \frac{1}{4\pi\varepsilon_o}\frac{q}{r_A}$$

In general form;

$$V = \frac{1}{4\pi\varepsilon_o} \cdot \frac{q}{r}$$

This is mathematical expression for electric potential.

Electron Volt:

Amount of energy gained or lost by an electron as it moves in a potential difference of one volt.

Mathematically:

Electrical Energy $= qV$

for an electron volt

$$1eV = e(1V)$$
$$= (1.6\times10^{-19}C)(1V)$$

$= 1.6 \times 10^{-19}$ CV

$1 eV = 1.6 \times 10^{-19} J$

Joule is the greater unit of energy as compared to electron volt.

Comparison Between Electric and Gravitation Forces:

Electric Force $= F_e = K \frac{q_1 q_2}{r^2}$

Gravitational Force $= F_g = G \frac{m_1 m_2}{r^2}$

Similarities:

i. Both the forces are conservative.

ii. Both the forces are natural.

iii. Both the forces are inversely proportional to the square of distance between charges or masses.

Electric Force	Gravitational Force
i- Electric force is for charges.	i- Gravitational force is for masses
ii- The electric constant is $K = 9 \times 10^{9} Nm^2 C^{-2}$	ii- The gravitational constant is: $G = 6.67 \times 10^{-11} Nm^2 Kg^{-2}$
iii- It is a strong force.	iii- It is a weak force.
iv- It may be attractive or repulsive	iv- It is only attractive.
v- It is medium dependent	v- It is independent of the medium.
vi- It can be sheilded.	vi- It can not be sheilded.

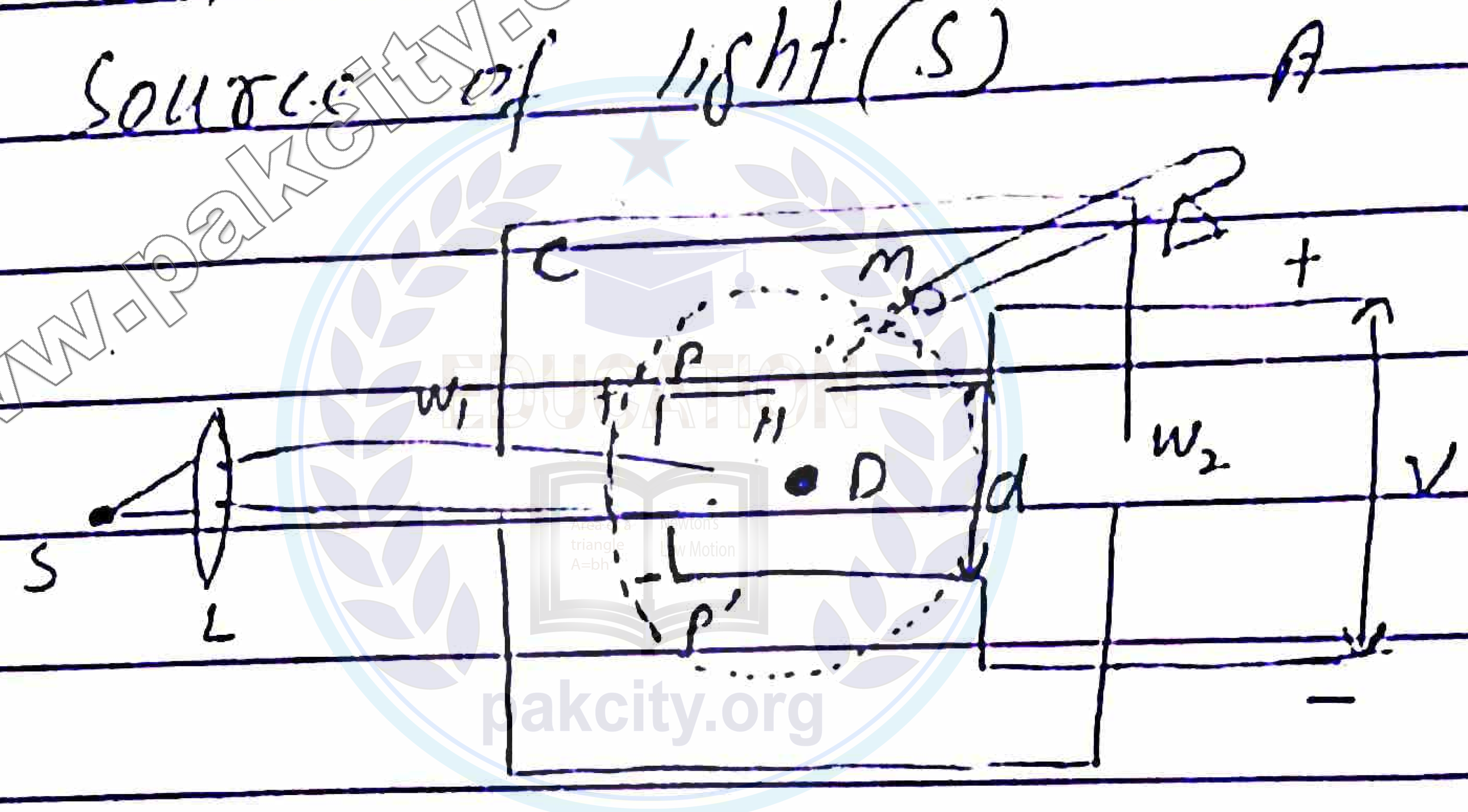
* Charge On An Electron By Millikan's Method:-

Introduction: In 1909 Robert Millikan conduct the oil drop experiment to determine the charge of an electron.

Construction: Millikan's method consists of the following parts:

i- Container (C)

ii- Source of light (S)



iii- Lens (L)

iv- Windows W_1 and W_2

v- Automizer (A)

vi- Parallel plates P and P'

vii- Hole in upper plate (H)

viii- Oil droplet (D)

ix- Microscope (M)

x- Distance between parallel plate (d)

xi- Potential difference (V)

Working:

The oil drops are sprayed through the automizer. Due to the friction of oil drops with the nozzle of automizer drops get charge. These charged oil drops fall down through the hole in upper plate. Then we apply electric force to balance the weight of oil droplet (D). This illuminated oil droplet is examined by the microscope (M).

Mathematical Expression:

To suspend the charged oil droplet we balance the weight of droplet with the electric force provided by the battery.

$$F_e = F_g$$

$$qE = mg$$

according to definition of potential gradient

$$E = \frac{V}{d}$$

So,

$$\frac{qV}{d} = mg$$

$$qV = mgd$$

$$q = \frac{mgd}{V}$$

By the above expression we can calulate the charge of the oil droplet if we know the values of terms at right side. The values of g, d and V are known and we have find the mass of droplet.

Determination of mass (m):

To determine the mass of the droplet, we switch off the applied potential difference. Now the droplet falls due to its weight, drag force also acts on it. Now

Drag force = weight of droplet

$$6\pi\eta r v_t = mg$$

$$\frac{6\pi\eta r v_t}{g} = m \longrightarrow (1)$$

By this expression we can find the mass of droplet. But here radius of droplet (r) is unknow. So, we have to calculate the radius.

Determination of radius (r):

we know that

$$\text{density} = \frac{\text{mass}}{\text{volume}}$$

mass = density x volume

$$m = \rho V$$

For spherical oil droplet

$$V = \frac{4}{3}\pi r^3$$

So,

$$m = \frac{4}{3}\pi r^3 \rho \rightarrow (2$$

By comparing eq. (1 and eq. (2

$$\frac{4}{3}\pi r^3 \rho = \frac{6\pi\eta r v_t}{g}$$

$$\frac{2r^2\rho}{3} = \frac{3\eta v_t}{g}$$

$$2r^2\rho = \frac{9\eta v_t}{g}$$

$$r^2 = \frac{9\eta v_t}{2\rho g}$$

$$r = \sqrt{\frac{9\eta v_t}{2\rho g}}$$

By this formula, we will calculate the radius of oil droplet, then we will find the mass of oil droplet and finally the charge of oil droplet

Result:

Millikans measured the charge on many drops and found that each charge was an integral multiple of a minimum value of charge equal to 1.6×10^{-19} C. i.e, therefore concluded that this minimum value of the charge is the charge on an electron.

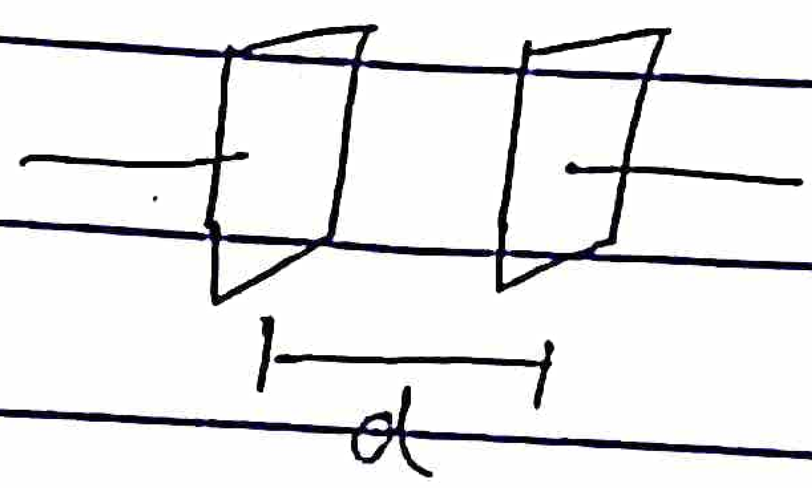
Capacitor

Defination:

A capacitor is a device which is used to store charge.

Construction:

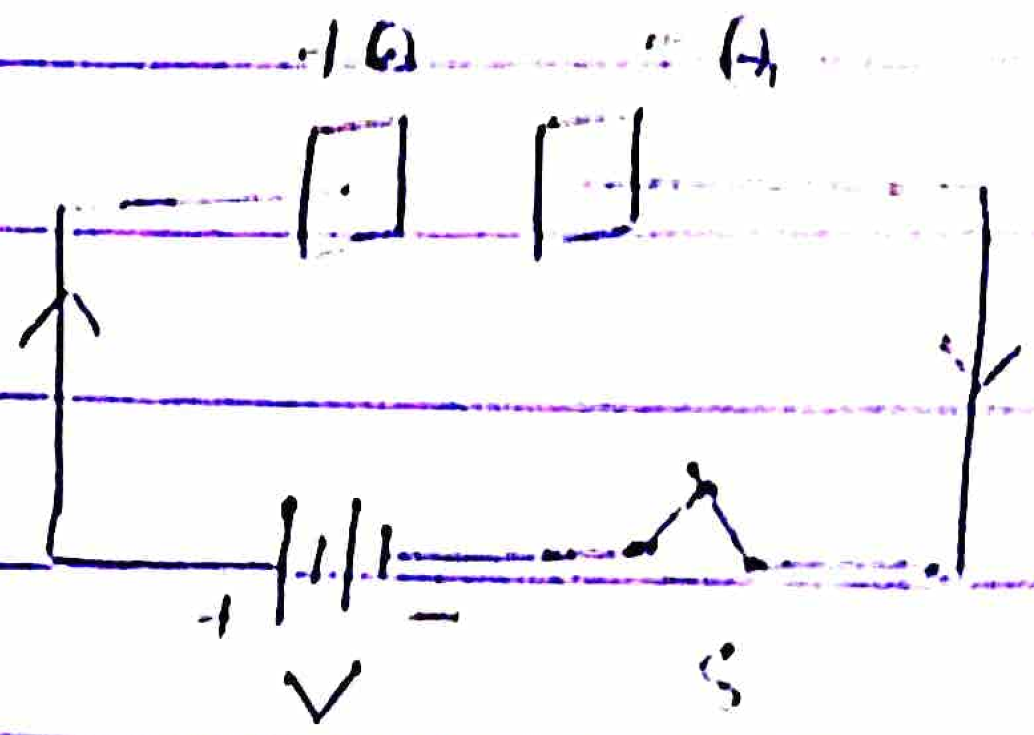
A parallel plate capacitor consists of two parallel plates of the conducting material. The plates are seperated by a small distance which contains air vacuum or some other insulating material.



Dielectric:

The insulating material present between the plates of the capacitor is called dielectric.

When the plates of the parallel plate capacitor are connected



to the terminal of the battery and switch is closed, a current starts flowing in the circuit. The plate connected to the positive terminal stores positive charge while the other plate connected to the negative terminal of battery stores negative charge. Let (Q) be amount of charge stored on the plates of capacitor and (V) is the potential difference of battery. Then

$$Q \propto V$$

$$Q = CV$$

Here (C) is the proportionality constant called capacitance of the capacitor.

Capacitance:

The ability of a capacitor to store charge is called its capacitance. It depends upon the geometry of the plates and medium between the plates.

The SI unit of capacitance is farad (F) named after English scientist Faraday.

Farad:

$$C = \frac{Q}{V}$$

$$1F = \frac{1C}{1V}$$

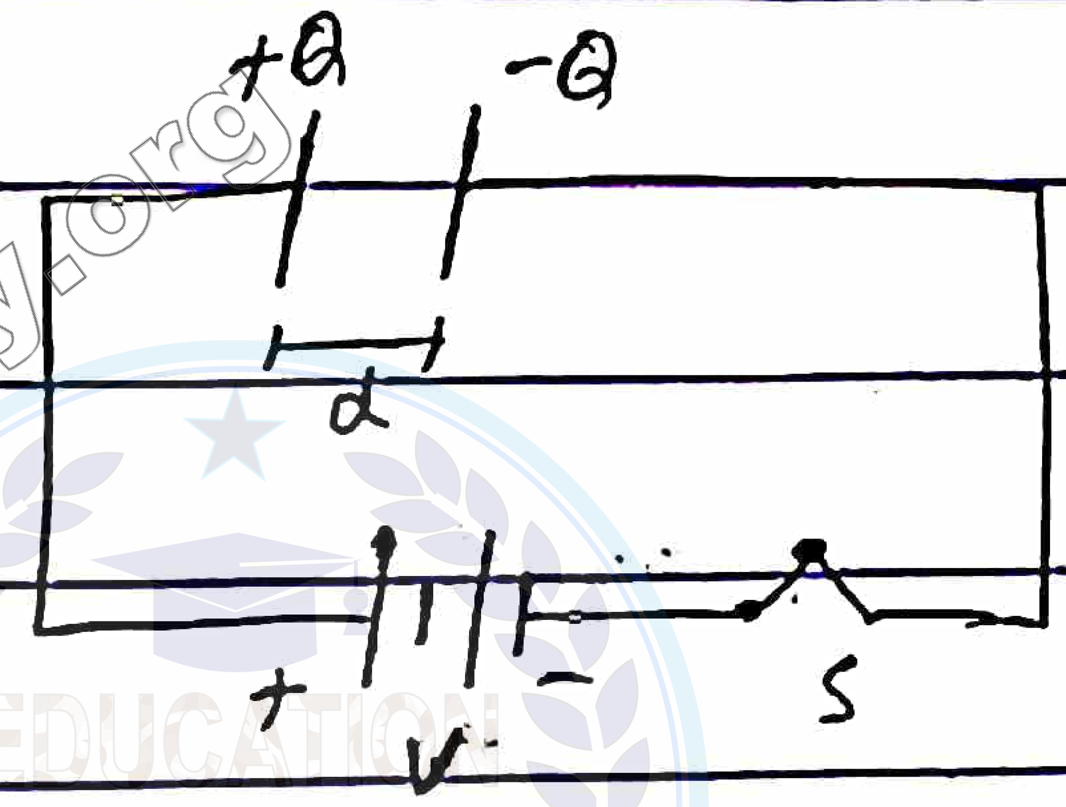
The capacitance of a capacitor is one Farad if a charge of one coulomb, given to one of the plates of the parallel plate capacitor, produces a potential difference of one volt between them.

Capacitance Of A Parallel Plate Capacitor:

Capacitance: The ability of a capacitor to store charge.
It depends on the geometry of the plates and medium b/w the Plates.

Explanation:

Consider a parallel plate capacitor having area A of the plates and distance d between the plates. The capacitor is connected to a battery of potential difference V as shown in the figure.



we know that the capacitance of the capacitor:

$$C_{vac} = \frac{Q}{V}$$

According to the defination of potential gradient.

$$E = \frac{V}{d} \longrightarrow (1$$

According to the Gauss's law; for two parallel charged plates:

$$E = \frac{\sigma}{\varepsilon_o}$$

Here σ is the surface charge density, given by:

$$\sigma = \frac{Q}{A}$$

So,

$$E = \frac{Q}{A\varepsilon_o} \longrightarrow (2$$

Comparing eq.(1 and eq.(2

$$\frac{Q}{A\varepsilon_o} = \frac{V}{d}$$

$$\frac{Q}{V} = \frac{A\varepsilon_o}{d}$$

$\frac{Q}{V}$ is the expression

for the capacitance of the capacitor. So,

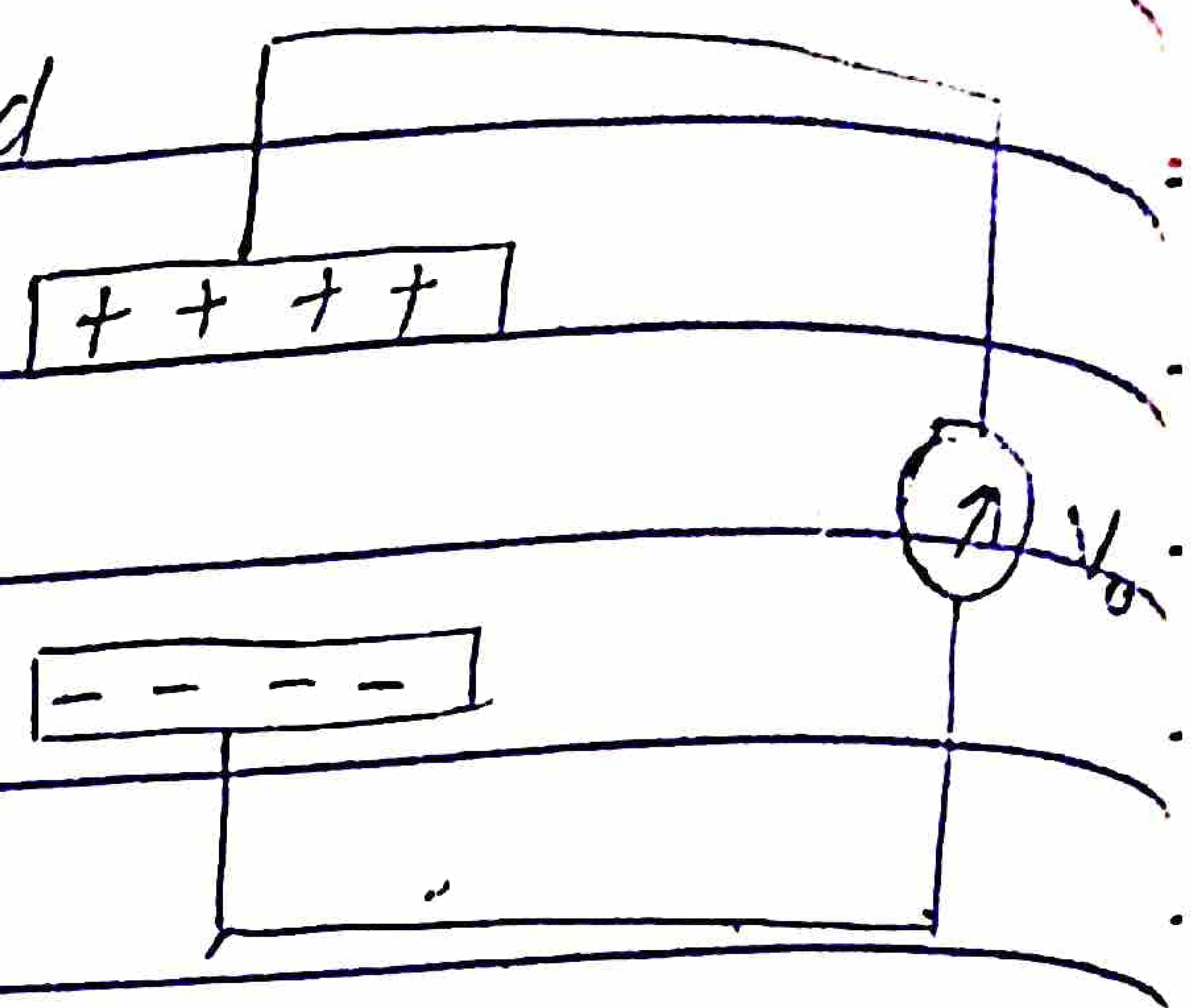
$$C_{vac.} = \frac{A\varepsilon_o}{d}$$

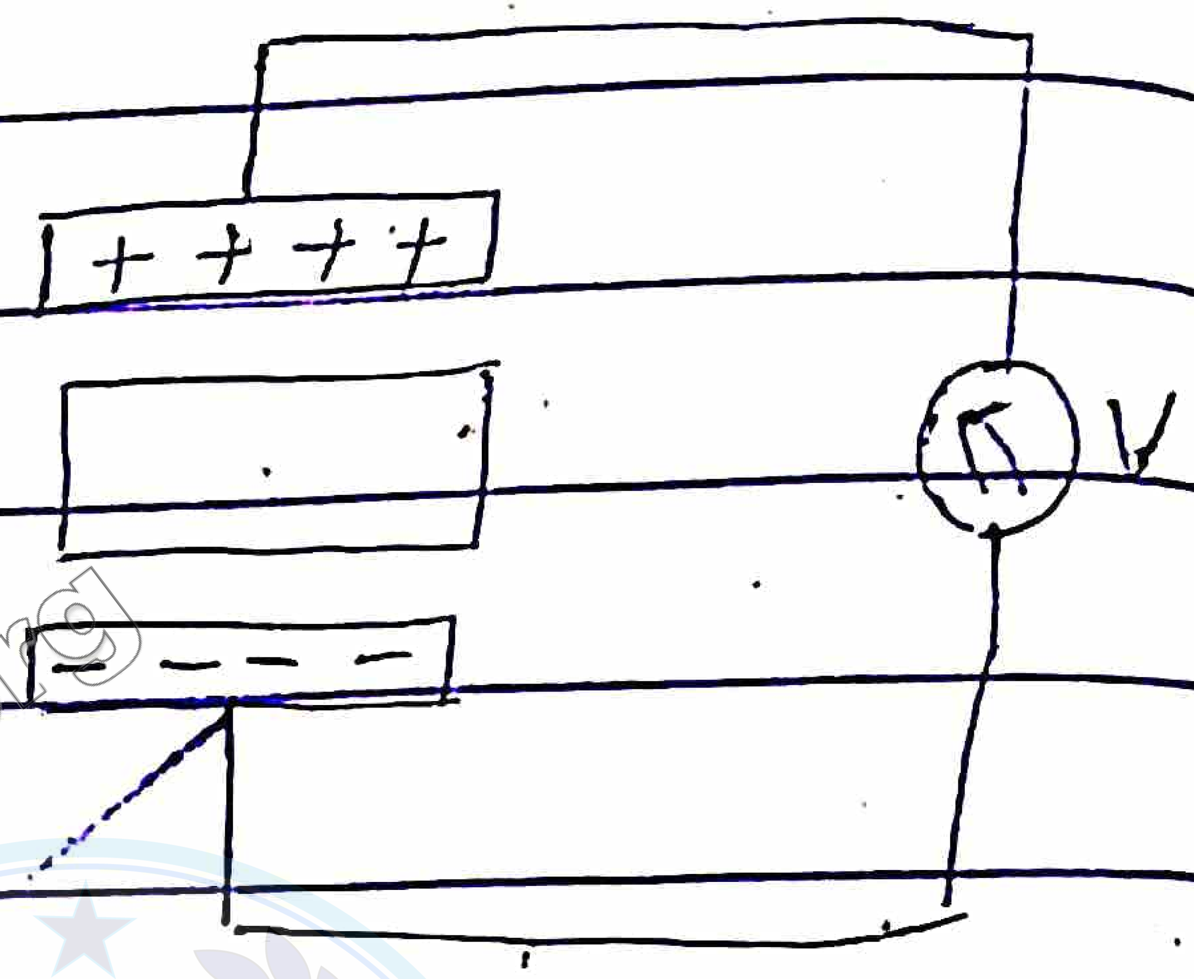
This is the expression for the capacitance of a capacitor when there is no medium between the plates. It shows that capacitance is directly proportional to the area of plates and inversly proportional to the distance between the plates.

Effect of medium:

Due to a dielectric medium between the plates of the capacitor, the capacitance of the capacitor increases by a factor ε_r. This fact can be explained by the following experiment.

A charged capacitor is connected to a voltmeter and we measure the potential difference V_0 between the plates of the capacitor.





Then we place a dielectric medium between the plates and again measure the potential difference. we see that the potential difference is decreases due to dielectric. So, according to relation $C = \frac{Q}{V}$ the capacitance is increased. mathematically,

$$C_{med} = \frac{A \epsilon_o . \epsilon_r}{d}$$

Here ε_r is the dielectric coefficient or dielectric constant.

Dielectric Co-efficient:

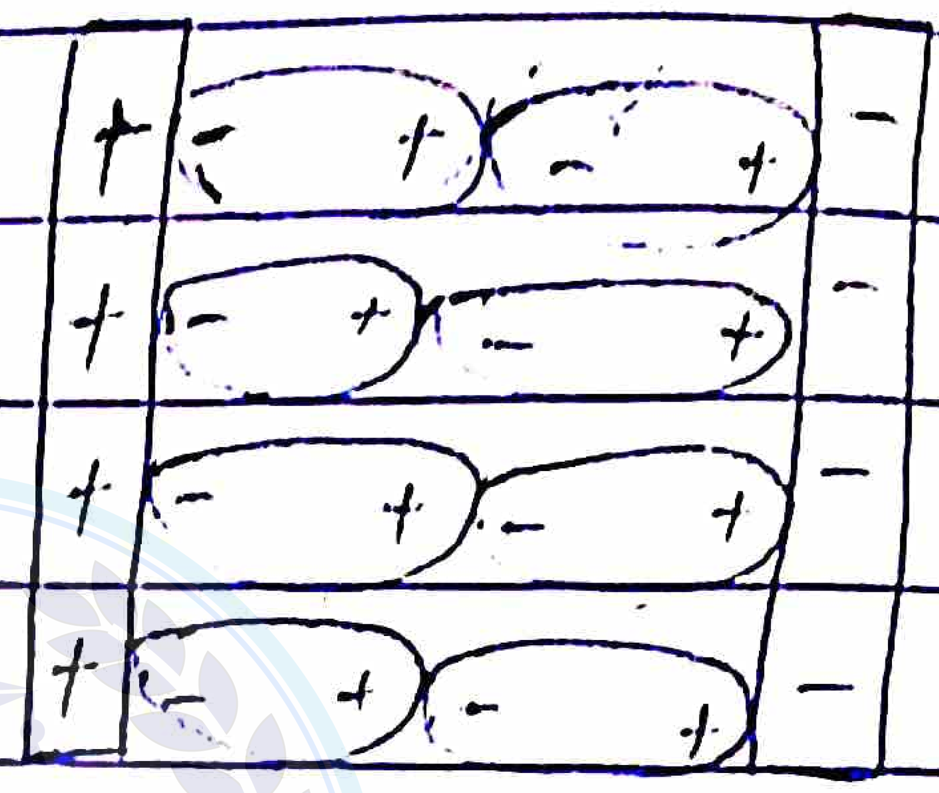
$$\frac{C_{med}}{C_{vacc}} = \frac{\frac{A\varepsilon_o\varepsilon_r}{d}}{\frac{A\varepsilon_o}{d}}$$

$$\frac{C_{med}}{C_{vacc.}} = \varepsilon_r$$

The ratio of the capacitance of a parallel plate capacitor with an insulating substance as medium between the plates to its capacitance with vaccume as medium between the plates.

Dipole:
Two equal and opposite charges seperated by a small distance form a dipole.

Electric Polarization of Dielectrics:
When a dielectric medium is placed between the plates of a charged capacitor, dipoles are formed.



So, the surface charge density upon the plates decreases. The positive charges are attracted towards negative plate and negative charges are attracted towards positive plate. Due to decrease in surface charge density σ, the electric intensity decreases $E = \frac{\sigma}{\varepsilon_0}$. The potential difference also decreases.

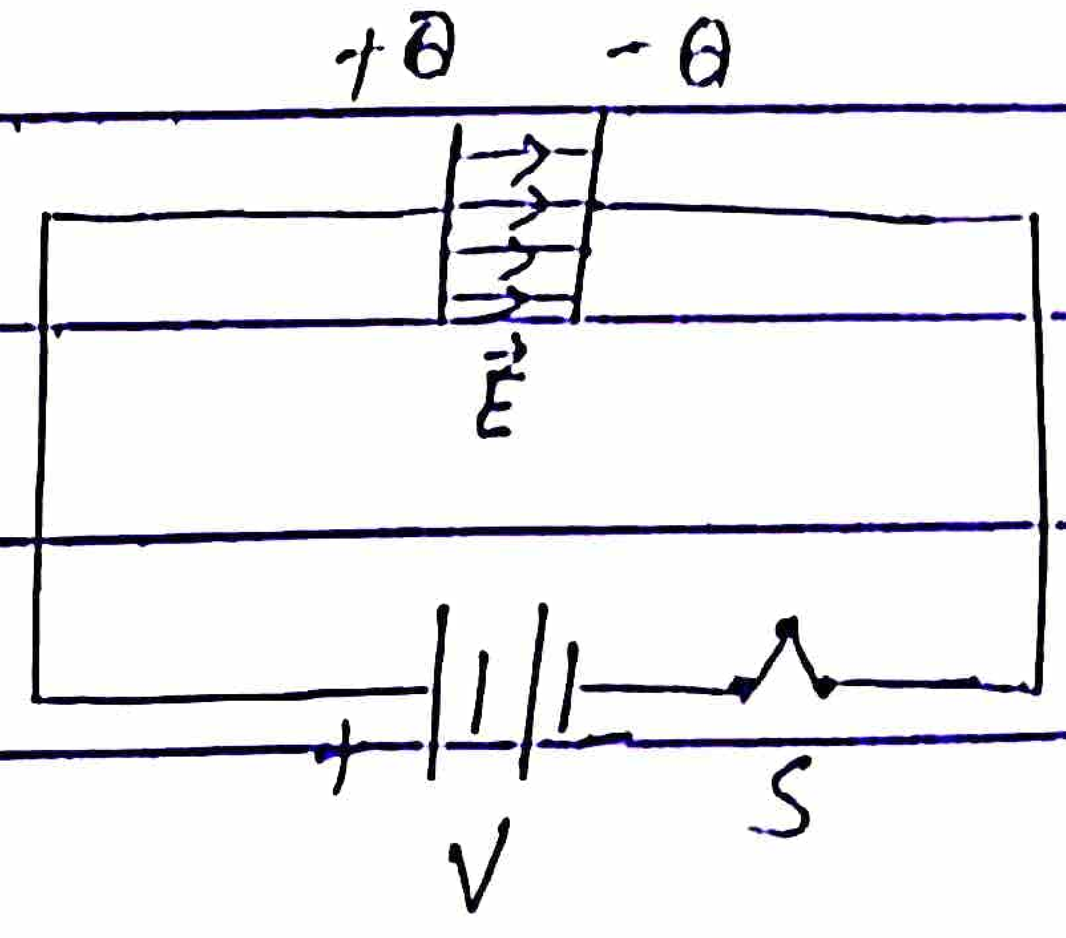
Finally the capacitance $C = \frac{Q}{V}$ increases due to decrease in potential difference.

Energy Stored In A Capacitor:

A capacitor is a device which is used to store charge. The charge is stored in the form of electric field between the plates or on the plates. So we can say that capacitor stores electrical energy.

Explanation:

When the plates of capacitor are connected to the terminals of the battery. The plate towards the



positive terminal of battery stores positive charge and the other plate connected to negative terminal stores negative charge. Initially the potential difference between the plates is zero and it becomes V after the charging. Average potential difference will be

$$\Delta V = \frac{0+V}{2} = \frac{V}{2}$$

If q is the amount of charge stored on the plates. Then

Electrical energy = $q \Delta V$

$$Energy = q\left(\frac{V}{2}\right)$$

$$Energy = \frac{1}{2} qV$$

we know that

$$q = CV$$

So,

$$Energy = \frac{1}{2}(CV)(V)$$

$$\text{Energy} = \frac{1}{2} C V^2$$

As $C = \frac{A \varepsilon_0 \varepsilon_r}{d}$, $E = \frac{V}{d}$

$\Rightarrow \quad V = Ed$

So,

$$\text{Energy} = \frac{1}{2} \left(\frac{A \varepsilon_0 \varepsilon_r}{d} \right) (Ed)^2$$

$$= \frac{1}{2} \left(\frac{A \varepsilon_0 \varepsilon_r}{\cancel{d}} \right) (E^2 d^{\cancel{2}})$$

$$\text{Energy} = \frac{1}{2} (\varepsilon_0 \varepsilon_r E^2)(Ad)$$

$$= \frac{1}{2} \varepsilon_0 \varepsilon_r E^2 (\text{Volume})$$

$$\frac{\text{Energy}}{\text{Volume}} = \frac{1}{2} \varepsilon_0 \varepsilon_r E^2$$

$$\text{Energy density} = \frac{1}{2} \varepsilon_0 \varepsilon_r E^2$$

Charging and discharging a Capacitor.

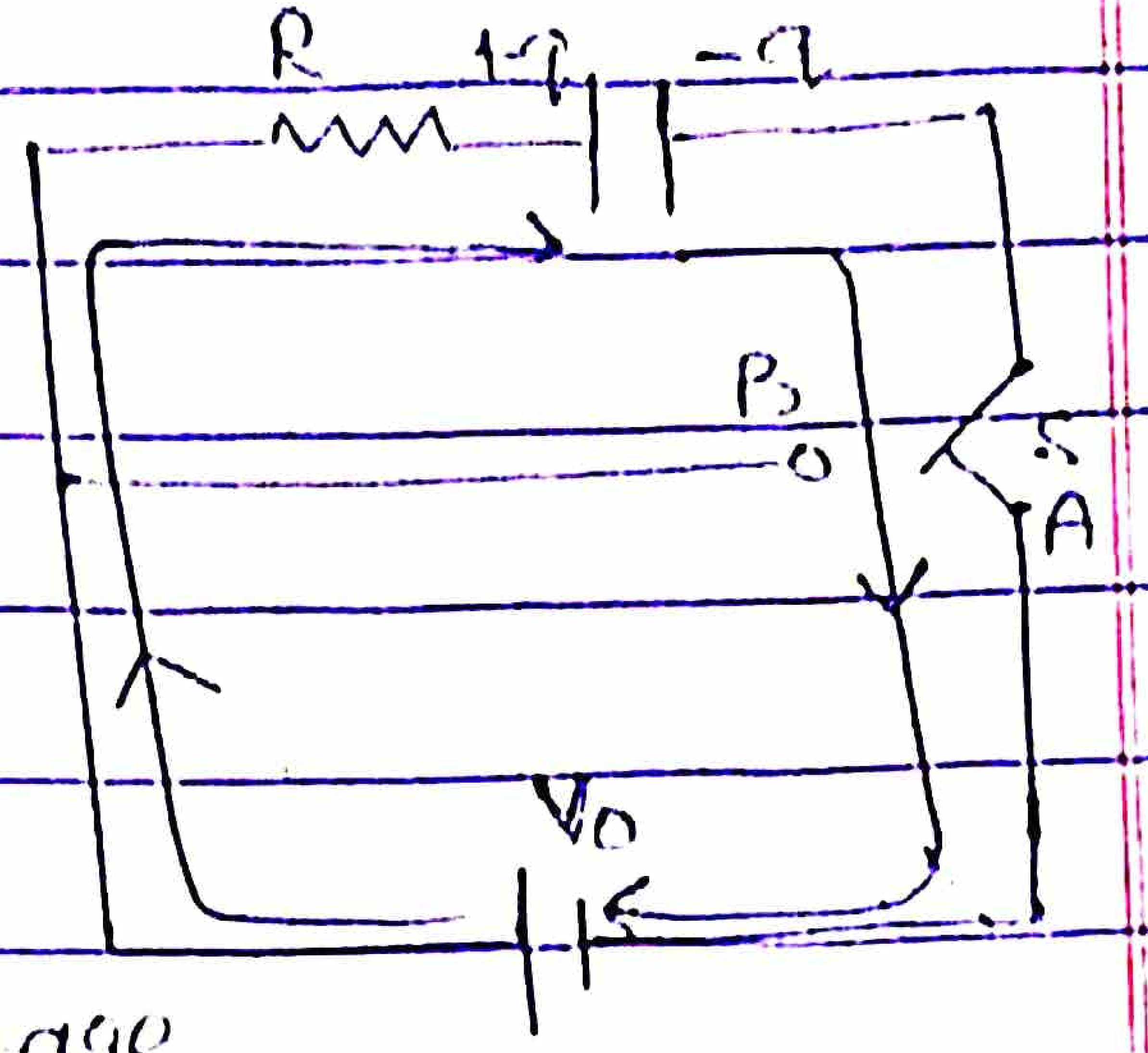
Rc-circuit:

Many electric circuits consist of both capacitors and resistors called resistor, capacitor circuit.

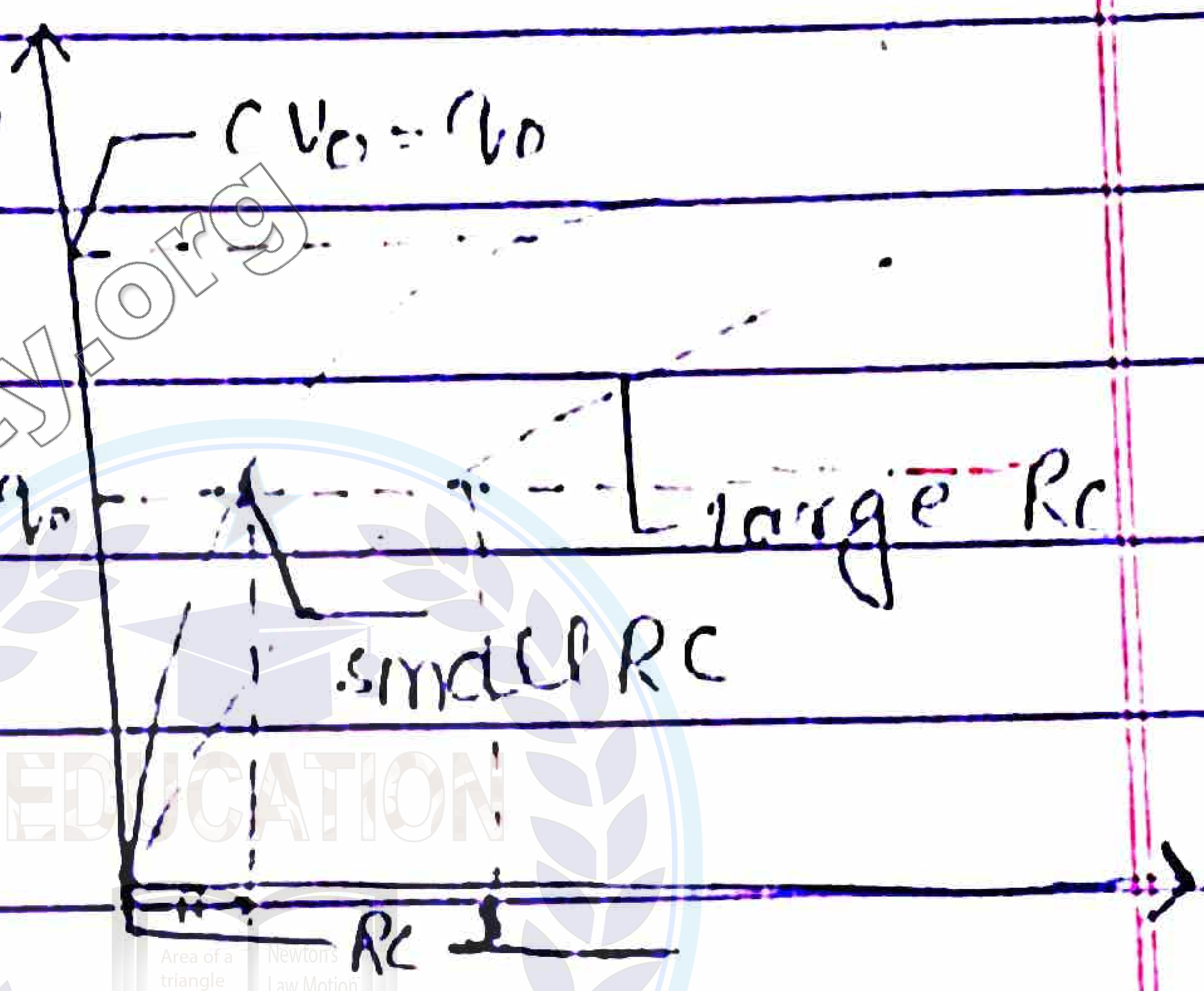
Time Constant:

How fast or how slow the capacitor is charging or discharging, depends upon the product of the resistance R and the capacitance C used in the circuit. As the unit of product RC is that of time, so this product is known as time constant and is defined as the time required by the capacitor to deposit 0.63 times the equilibrium charge q_o.

Changing of Capacitor:



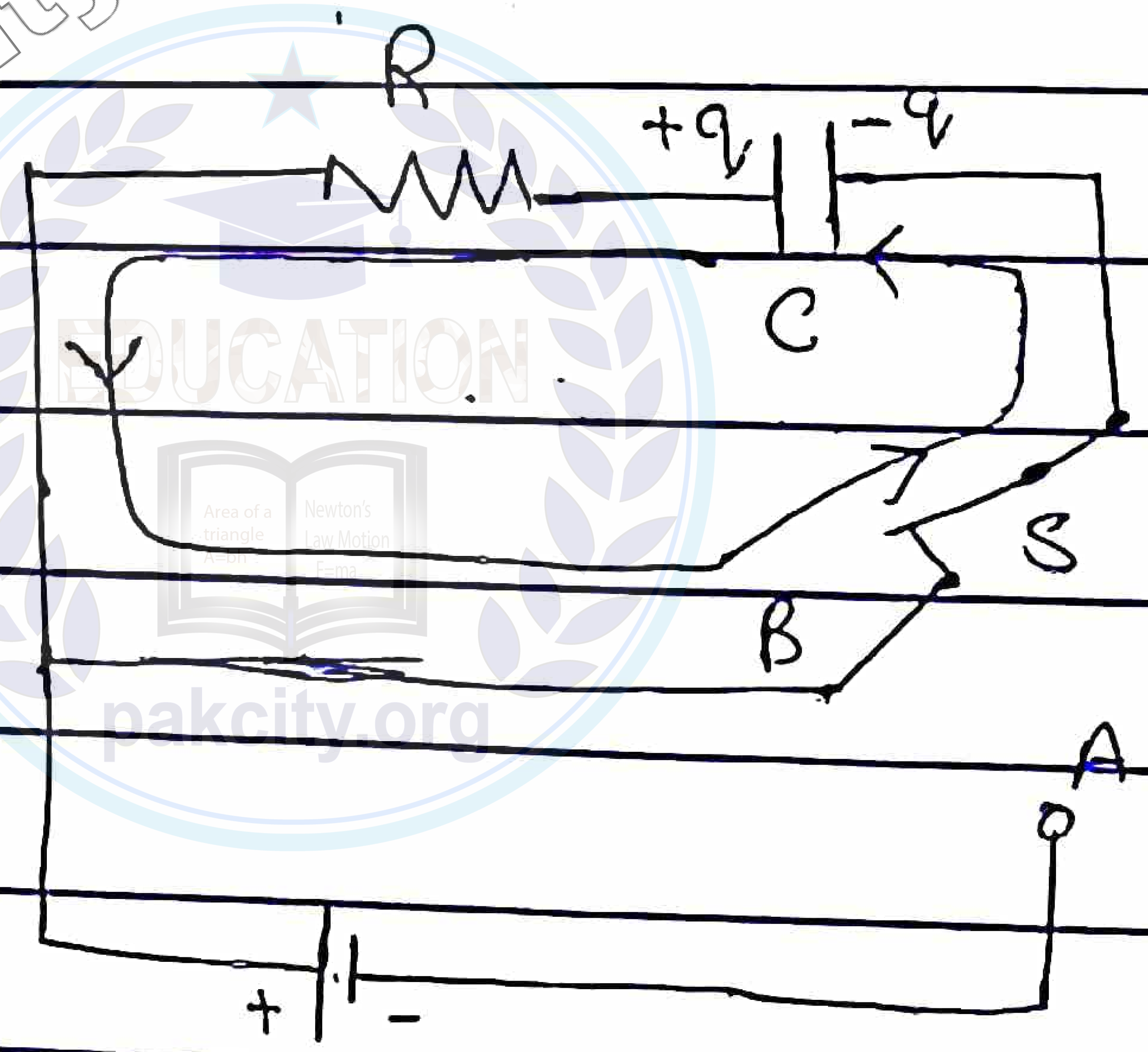
When the switch S is set as terminal A the R-C combination is connected to a battery of voltage V_0 which starts charging the capacitor through the Resistor R. The capacitor is not charged immediately, rather charges build up gradually to the equilibrium value of $q_0 = CV_0$. The growth of charge with time ~~till it reaches~~ for different resistances is shown in fig. According to this graph $q = 0$ at $t = 0$ and increases gradually with



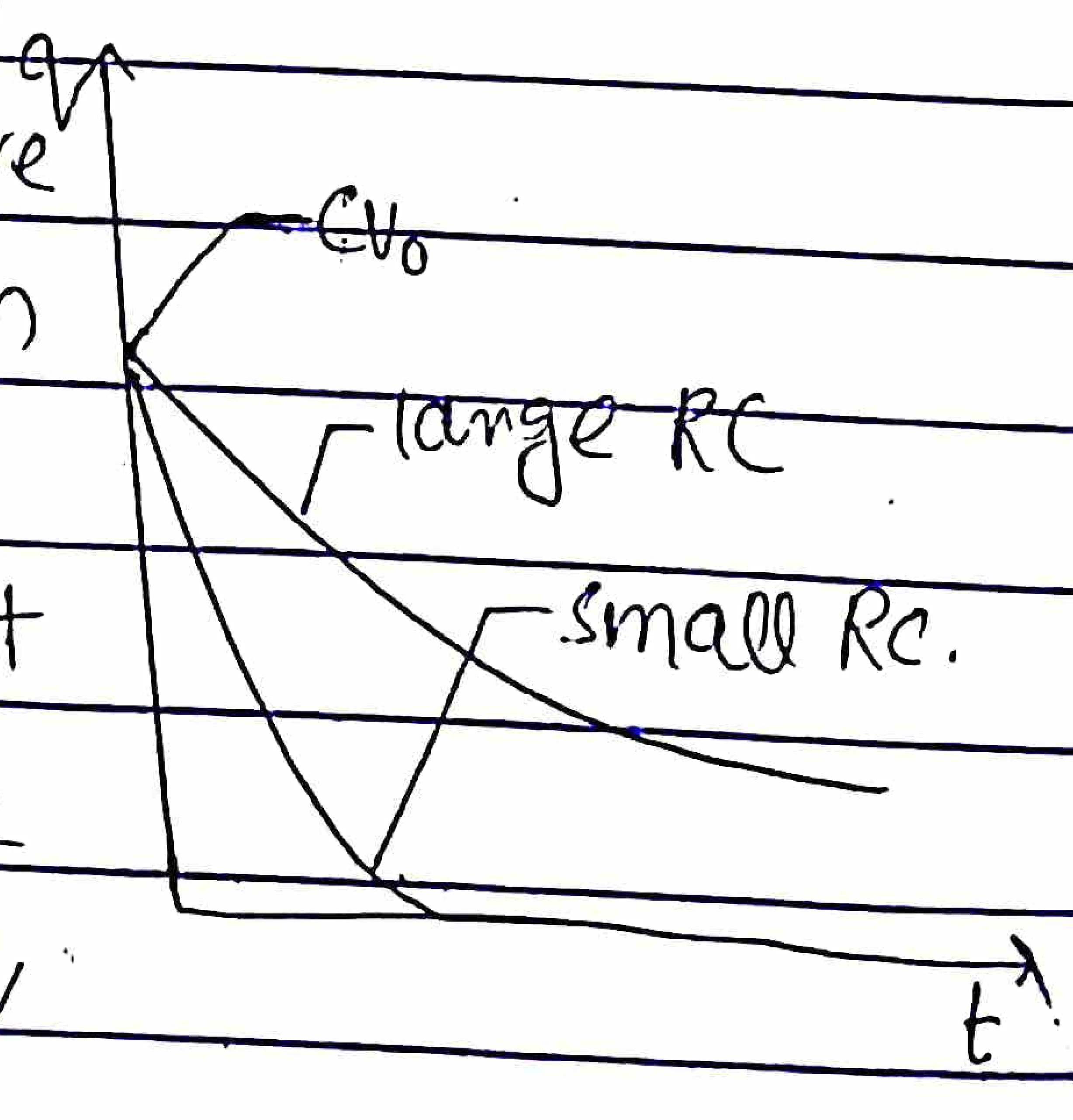
time till it reaches its equilibrium value $q_0 = CV_0$. This voltage V across capacitor at any instant can be obtained by dividing q by C, as $V = q/C$.

Discharging of capacitor:

Figure illustrates the discharging of a capacitor through a resistor.



In this figure the switch S is set at point B so the charge +q



on the left plate can flow anti-clock wise through the resistance and neutralize the charge $-q$ on the right plate.

The graph in fig shows that discharging begins at $t=0$ when $q=CV_0$ and decreases gradually to zero. Smaller values of time constant RC lead to a more rapid discharge.

12.1: The negative of potential gradient will be equal to electric intensity.

$$E = -\frac{\Delta V}{\Delta r}$$

If the potential is constant.

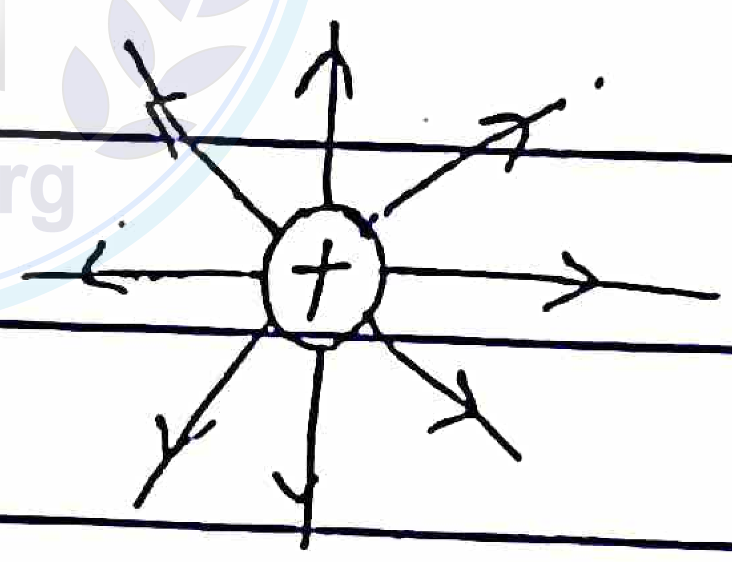
$$V = \text{constant}$$

$$\Delta V = 0$$

So, $E = 0$

Therefore, the electric field is zero in the region where the potential is constant.

12.2 If we follow the electric field line due to a positive point charge,



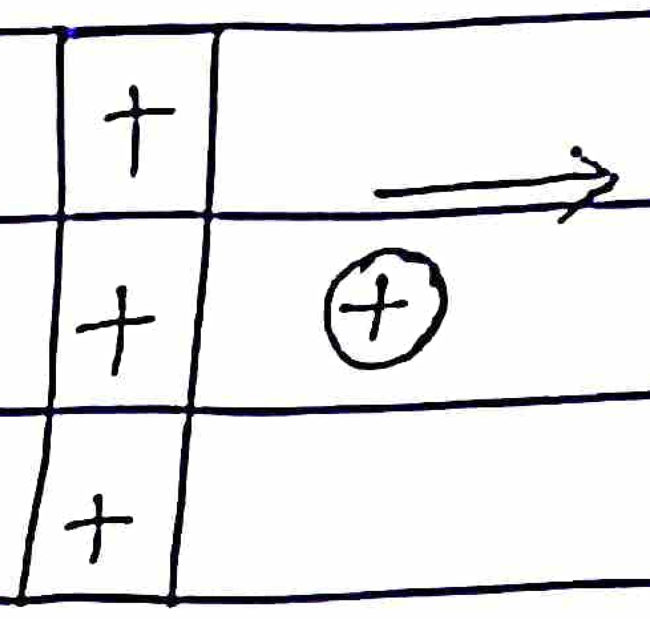
we will move away from the charge. So, the electric field and potential will decrease.

Mathematically, electric field intensity and electric potential are inversly proportional to the distance.

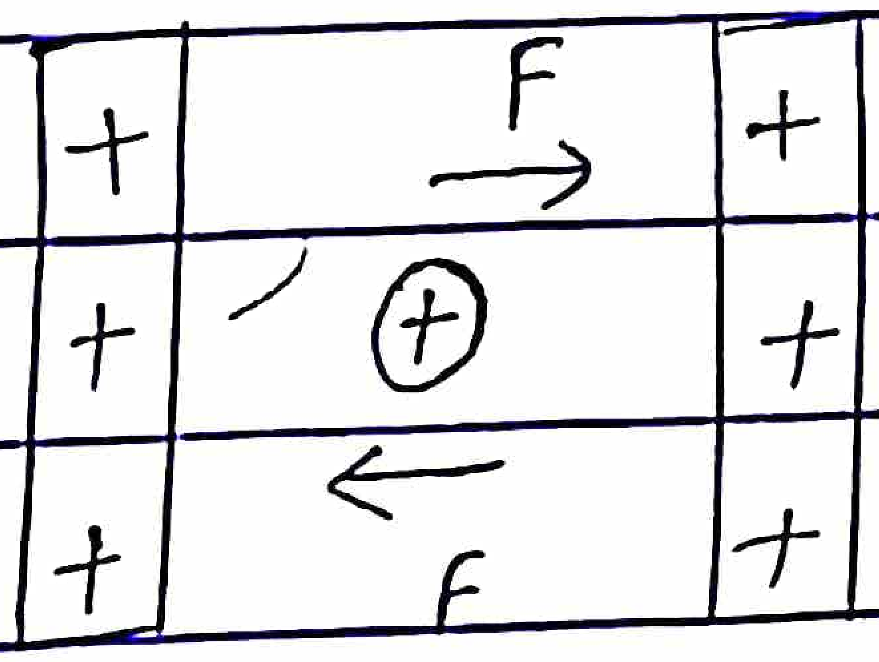
$$E = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2} \quad \text{and} \quad V = \frac{1}{4\pi\varepsilon_0}\frac{q}{r}$$

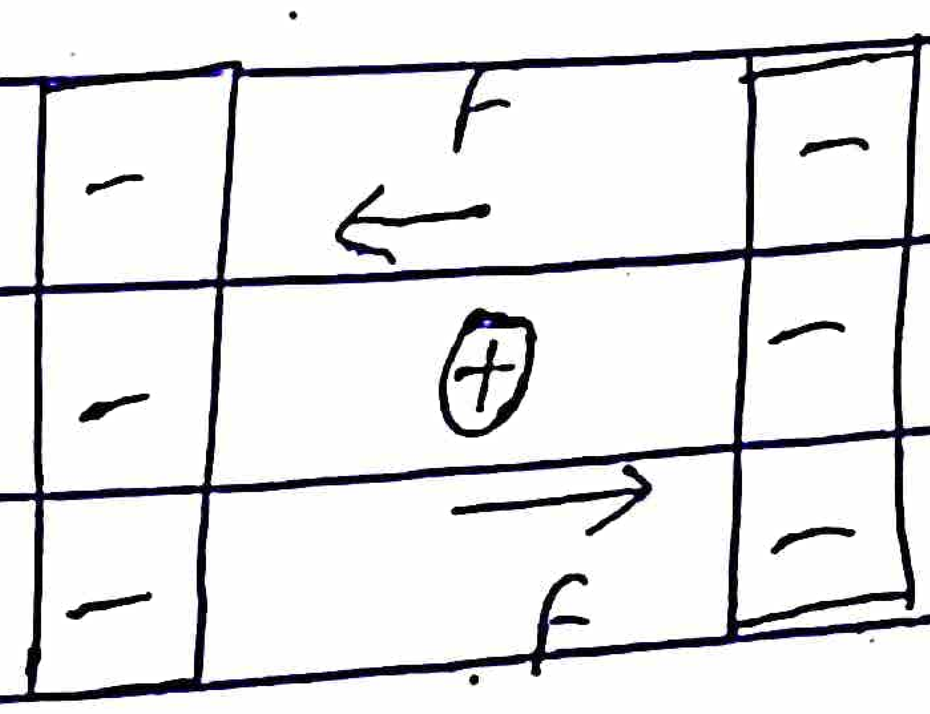
So, when r increases E and V decreases

12.3 To identify the positively charged plate of a capacitor we will use a positive test charge. The positively charged plate will repel the test charge and identified.



12.4 (a) The net force will be zero on the charge when it is placed between parallel plates with



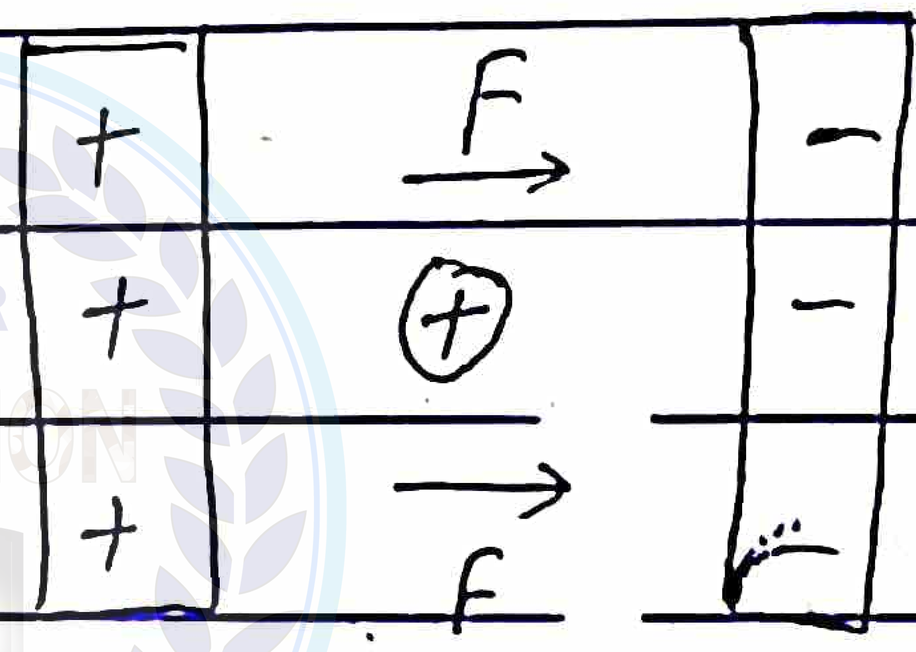


similar and equal charges. Because the positive point charge will be attracted or repelled by the opposite plates. resulting zero net force.

Net force = F − F

Net force = 0

(b) The net force will be 2F in this case.

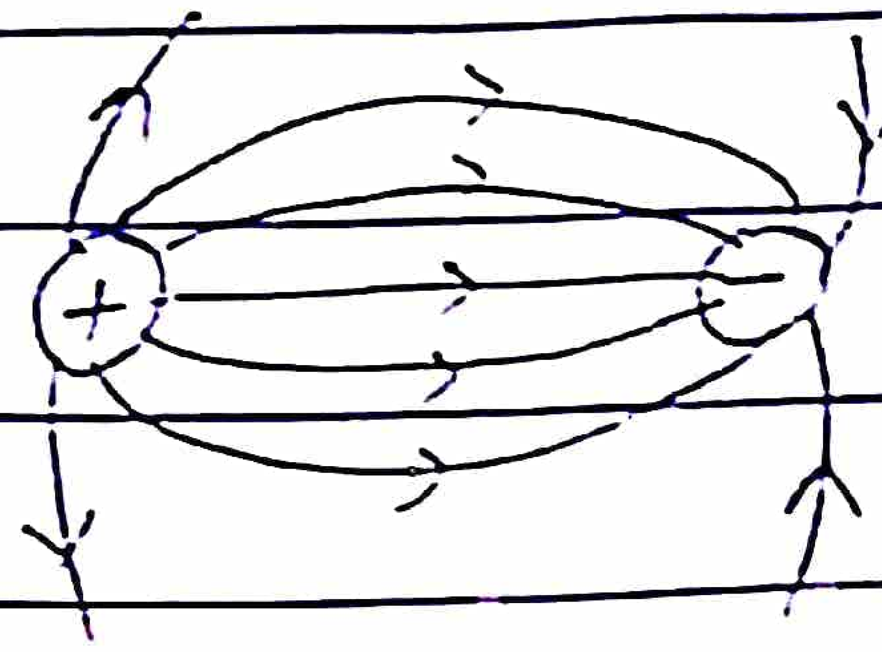


Because when the positive point charge is placed between parallel plates with opposite and equal charges it will be repelled by positive plate and attracted by negative plate. So

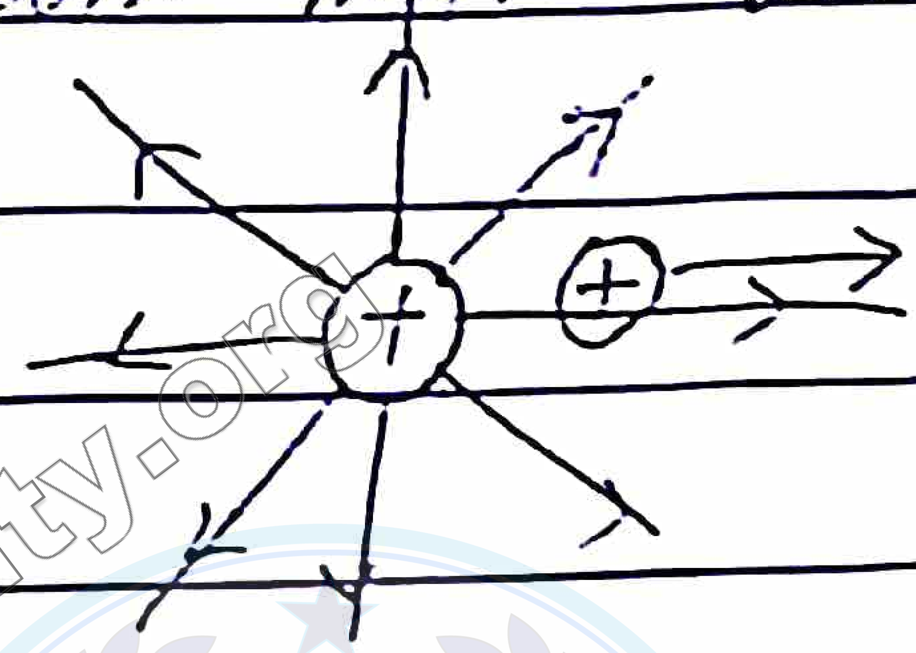
Net force = F + F

Net force = 2F

12.5 Electric lines of force never cross because each line has its own specific direction.



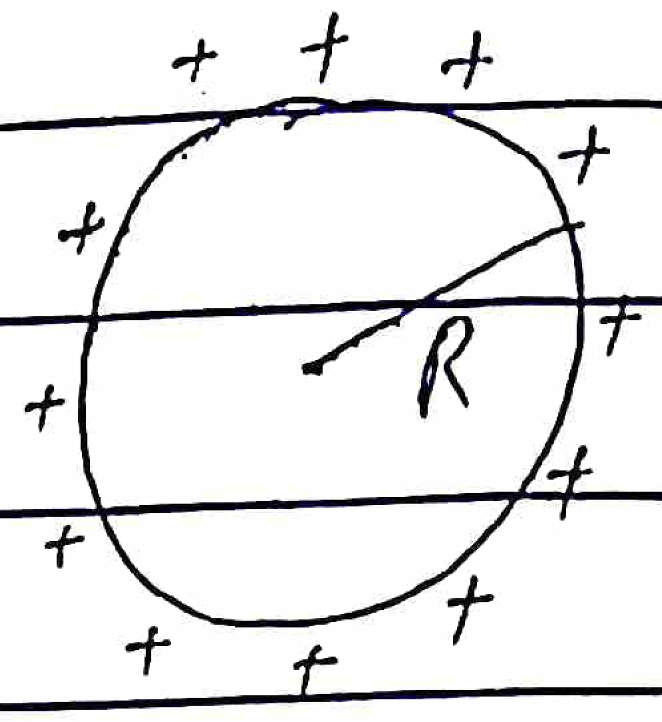
12.6 Yes, it will make rectilinear motion.



The motion of any object in a straight line is called rectilinear motion. If a point charge q of mass m is released in a non-uniform electric field it will be repelled and move in a straight line.

12.7 Yes, $\vec{E}$ is necessarily zero inside a charged rubber balloon if the balloon is spherical.

According to Gauss's law



$$\phi = \frac{1}{\varepsilon_0}(Q)$$

There is no charge inside the balloon. So, $Q = 0$

$$\phi = \frac{1}{\varepsilon_0}(0)$$

$$\phi = 0$$

And $\phi = \vec{E} \cdot \vec{A}$

comparing the above equations;

$$\vec{E} \cdot \vec{A} = 0$$

either $\vec{E} = 0$ or $\vec{A} = 0$

But area can not be zero. $\vec{A} \neq 0$

Therefore,

$$\vec{E} = 0$$

12.8 Yes, it is true that Gauss's law states that the total number of lines of forces crossing the closed surface in the outward direction is proportional to the net positive charge

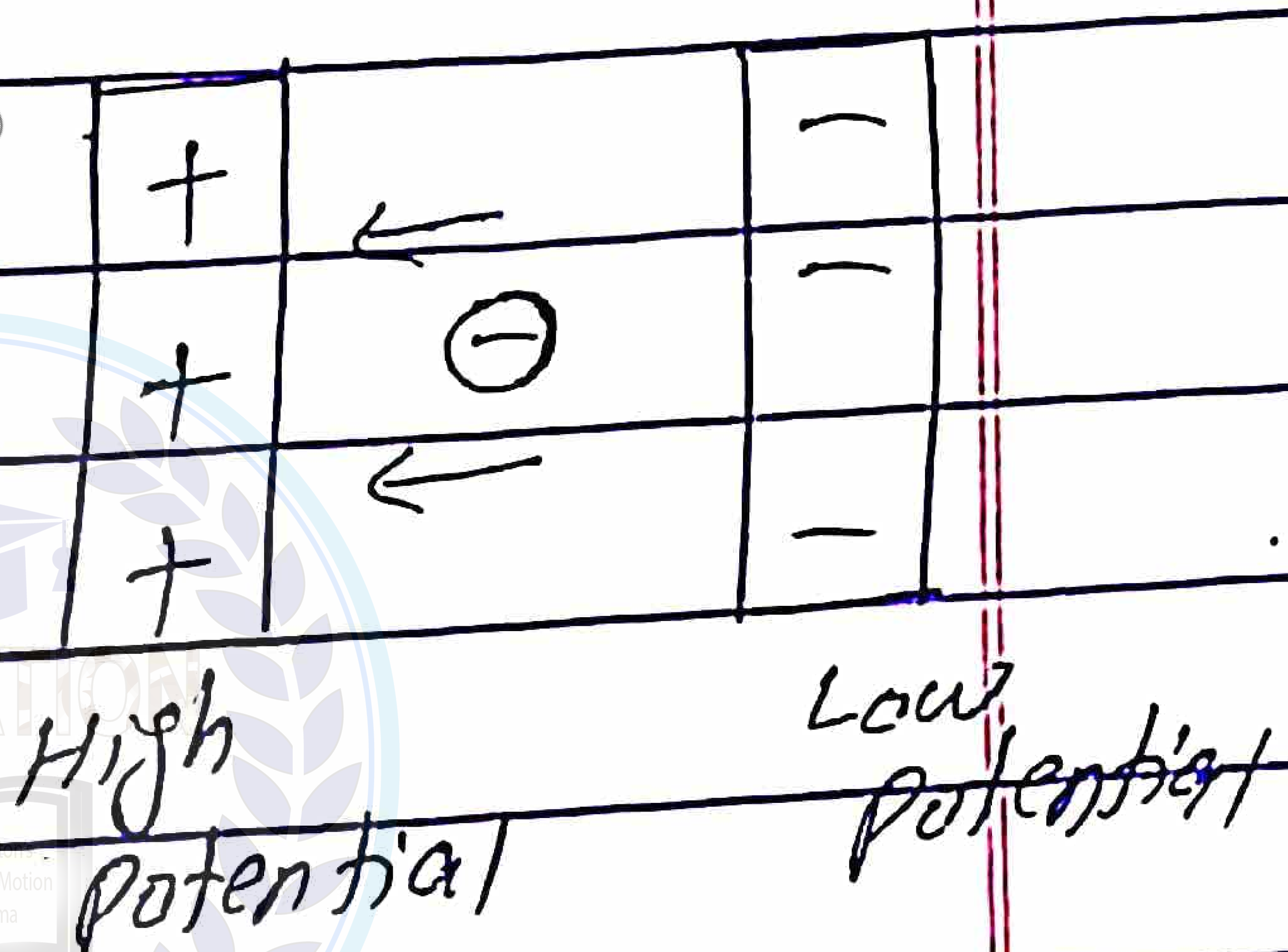
enclosed within the surface.
According to Gauss's law:

$$\Phi = \frac{1}{\varepsilon_0}(Q)$$

$$\Phi \propto Q$$

Number of lines of force $\propto$ Net positive charge

12.9 Electron has negative charge. In the electric field, it will move from negative end towards positive end.



Therefore, it will move from low potential to high potential.

12.1 Data

$\frac{F_e}{F_g} = ?$, masses of objects $= m_1 = m_2$

$m_1 = m_2 = 10g = \frac{10}{1000} kg = .001 kg$

charge for objects $= q_1 = q_2 = 20 \mu C$

$q_1 = q_2 = 20 \times 10^{-6} C$

distance between objects $= r = 10cm$

$r = \frac{10}{100} m = 0.1 m$

gravitational constant $= G = 6.67 \times 10^{-11} Nm^2 kg^{-2}$

coulomb constant $= K = 9 \times 10^9 Nm^2 C^{-2}$

Solution:

$$F_e = K \frac{q_1 q_2}{r^2}$$

$$= \frac{9 \times 10^9 \times 20 \times 10^{-6} \times 20 \times 10^{-6}}{(0.1)^2}$$

$$F_e = 360 N$$

$$F_g = G \frac{m_1 m_2}{r^2}$$

$$= \frac{6.67 \times 10^{-11} \times 0.01 \times 0.01}{(0.1)^2}$$

$$F_g = 6.67 \times 10^{-13} N$$

$$\frac{F_e}{F_g} = \frac{360}{6.67\times10^{-13}}$$

$$\boxed{\frac{F_e}{F_g} = 5.4\times10^{14}}$$

12.3 Data

point charge $= q = -8\times10^{-8}$ C

electric field $= E = ?$

distance from origin $= r = 2$ m

coulomb's constant $= k = 9\times10^{9}\ Nm^2C^{-2}$

Solution: $E = k\frac{q}{r^2}$

$$= \frac{9\times10^{9}\times -8\times10^{-8}}{(2)^2}$$

$$E = -180\ NC^{-1}$$

$$E = -1.8\times10^{2}\ NC^{-1}$$

As $\vec{E}$ acts along z-axis.

$$\boxed{\vec{E} = (-1.8\times10^{2}\hat{k})\ NC^{-1}}$$

12.4 Data

electric field $= \vec{E} = ?$

position vector $= \vec{r} = (4\hat{i} + 3\hat{j})\,m$

point charge $= q = 5 \times 10^{-6}\,C$

coulomb's constant $= k = 9 \times 10^{9}\,Nm^2C^{-2}$

Solution:

$$\vec{E} = k\frac{q}{r^2}\hat{r}$$

Given $\vec{r} = 4\hat{i} + 3\hat{j}$

$$r = \sqrt{(4)^2 + (3)^2} = \sqrt{16+9}$$

$$r = \sqrt{25}$$

$$r = 5\,m$$

Now

$$\vec{E} = k\frac{q}{r^2}\left(\frac{\vec{r}}{r}\right)$$

$$= \frac{kq}{r^3}\vec{r}$$

$$= \frac{9 \times 10^{9} \times 5 \times 10^{-6}}{(5)^3}(4\hat{i} + 3\hat{j})$$

$$= 360(4\hat{i} + 3\hat{j})$$

$$\boxed{\vec{E} = (1440\hat{i} + 1080\hat{j})\,NC^{-1}}$$

12.5 Data

1st point charge $= q_1 = -1 \times 10^{-6}\ C$

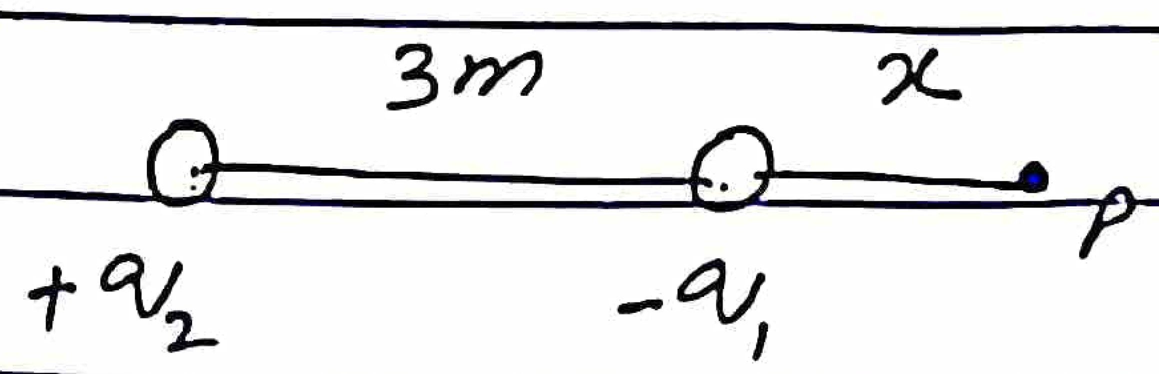
2nd point charge $= q_2 = 4 \times 10^{-6}\ C$

distance between charges $= r = 3\ m$

Zero field location $= x = ?$

Solution:

Suppose point P shows the zero field location. At



this point the electric field for both charge will be equal and opposite.

$$|E_1| = |E_2|$$

$$\left| K \frac{q_1}{r^2} \right| = \left| K \frac{q_2}{r^2} \right|$$

$$\left| K \frac{-1 \times 10^{-6}}{x^2} \right| = \left| K \frac{4 \times 10^{-6}}{(3+x)^2} \right|$$

$$\cancel{K} \frac{1 \times \cancel{10^{-6}}}{x^2} = \cancel{K} \frac{4 \times \cancel{10^{-6}}}{(3+x)^2}$$

$$4x^2 = (3+x)^2$$

$$\sqrt{4x^2} = \sqrt{(3+x)^2}$$

$$2x = 3 + x$$

$$2x - x = 3$$

$$\boxed{x = 3\,m}$$

Therefore, the zero field location is 3m from q_1.

12-6 Data

electric field strength $= E = ?$

mass of particle $= m = 1 \times 10^{-6}$ kg

charge of particle $= q = 1\,\mu C$

$$q = 10^{-6}\,C$$

distance between plates $= d = 10$cm

$$d = \frac{10}{100}\,m = 0.1\,m$$

$$g = 9.8\,ms^{-2}$$

Solution: To suspend the particle the weight should be equal to the electric force.

$$F_e = W$$

$$qE = mg$$

$$E = \frac{mg}{q}$$

$$E = \frac{1\times\cancel{10^{-6}}\times 9.8}{\cancel{10^{-6}}}$$

$$\boxed{E = 9.8\ NC^{-1}}$$

12.7 Data

charge of particle = q = 20 e

potential difference = V = 100 V

Energy = ?

Solution:

$$Energy = qV$$

$$= (20e)(100V)$$

$$Energy = 2000\ eV$$

$$\boxed{Energy = 2\times10^{3}\ eV}$$

12.10 Data

point charge = q = 4×10^{-8} C

electric potential = V = ?

distance = r = 1.2 m

$K = 9\times10^{9}\ Nm^{2}C^{-2}$

Solution:

$$V = k\frac{q}{r}$$

$$= \frac{9\times10^{9}\times4\times10^{-8}}{1.2}$$

$$V = 300\ V$$

$$\boxed{V = 3\times10^{2}\ V}$$

when the charge is negative.

$$\boxed{V = -3\times10^{2}\ V}$$

12.12 Data

capacitance $= C = 750\ \mu F$

$C = 750\times10^{-6}\ F$

potential difference $= V = 330\ V$

Energy = ?

Solution:

$$\text{Energy} = \frac{1}{2}CV^2$$

$$= \frac{1}{2}(750\times10^{-6})(330)^2$$

$$\boxed{\text{Energy} = 40.8\ J}$$

12.13 Data

capacitance $= C = 2.5 \times 10^{-8} F$

potential difference $= V = 450 V$

number of electrons $= n = ?$

charge of one electron $= e = 1.6 \times 10^{-19} C$

Solution: Total charge $= Q = ne$

we know that

$$Q = CV$$

Comparing the above equations

$$ne = CV$$

$$n = \frac{CV}{e}$$

$$n = \frac{2.5 \times 10^{-8} \times 450}{1.6 \times 10^{-19}}$$

$$\boxed{n = 7 \times 10^{13} \text{ electrons}}$$

12.11 Data

distance $= r = 5.29 \times 10^{-11} m$

speed $= v = 2.18 \times 10^{6} ms^{-1}$

charge of electron $= e = 1.6 \times 10^{-19} C$

mass of electron $= m = 9.1 \times 10^{-31} kg$

electric potential $= V = ?$, $K = 9 \times 10^{9} Nm^2 C^{-2}$

Total energy $= ?$

Ionization energy $= ?$

Solution:

$$V = K\frac{q}{r}$$

$$V = K\frac{e}{r}$$

$$= \frac{9\times10^{9}\times1.6\times10^{-19}}{5.29\times10^{-11}}$$

$$\boxed{V = 27.2\ V}$$

The total energy of the atom will be the sum of P.E and K.E.

$$K.E = \frac{1}{2}mv^2$$

$$= \frac{1}{2}(9.1\times10^{-31})(2.18\times10^{6})^2$$

$$K.E = 2.16\times10^{-18}\ J$$

As

$$1eV = 1.6\times10^{-19}\ J$$

$$K.E = \frac{2.16\times10^{-18}}{1.6\times10^{-19}}\ eV$$

$$K.E = 13.5\ eV$$

$$P.E = qV$$
$$= -eV$$
$$P.E = -e(27.2\,V)$$
$$P.E = -27.2\,eV$$
$$T.E = K.E + P.E$$
$$= 13.5 - 27.2$$
$$\boxed{T.E = -13.7\,eV}$$

To ionize the atom, the energy equal to this total energy should be provided. So,

$$\boxed{\text{Ionization energy} = 13.7\,eV}$$